Assignment #28

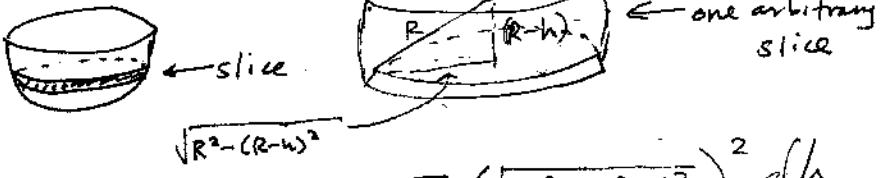
4/25/2001

27.1

(#13) a) Since $\rho(x)$ varies with the distance from the center of the truffle, we'll use concentric spherical shells.

$$\#\text{calories} = \int_0^R \rho(x) \underbrace{4\pi x^2 dx}_{\text{Vol. of slice}}$$

b) we must use horizontal slices



$$\text{Volume of each slice} = \pi\left(\sqrt{R^2-(R-h)^2}\right)^2 dh$$

$$\int_0^R \delta(x)\,\underbrace{\pi\left(\sqrt{R^2-(R-h)^2}\right)^2 dh}_{\text{Volume of slice}}$$

(#18) Since $\rho(r)$ depends on distance from the center, use concentric, spherical shells.

$$\text{Volume of slice} = 4\pi r^2 dr$$

$$\int_0^{90{,}000} \rho(r)\, 4\pi r^2 dr = \boxed{\int_0^{90{,}000} \frac{K}{(r+1)^{3/2}}\, 4\pi r^2 dr}$$

(#19) use cylindrical shells, since $\rho(x)$ depends on distance from axis

$$\text{Volume of Shell} = (2\pi r)(\text{height})\,dr$$

$$\text{mass} = \boxed{\int_0^3 \rho(x)\,(2\pi x)(4)\,dx}$$

(#21) Use vertical, cylindrical shells 

Volume of each shell: $(2\pi x)(\text{height})\,dx$

$$\boxed{\int_0^4 (2\pi x)\left(\frac{8}{1+\frac{x^2}{16}}\right)dx} = \text{Volume of tent}$$

$$16\pi\int_0^4 \frac{x}{1+\frac{x^2}{16}}dx = 16\pi\int_0^4 \frac{x}{\frac{16+x^2}{16}}dx = 16\pi\int_0^4 \frac{16x}{16+x^2}dx$$

$$= 256\pi\int_0^4 \frac{x}{16+x^2}dx \quad \begin{matrix} u = 16+x^2 \\ du = 2x\,dx \end{matrix} \Rightarrow \frac{256\pi}{2}\int_0^4 \frac{du}{u}$$

$$= 128\pi \ln|u|\Big|_{x=0}^{x=4} = 128\pi\left[\ln|16+x^2|\right]_0^4$$

$$= 128\pi\left[\ln 32 - \ln 16\right] = 128\pi(\ln 2)$$

$$\approx 279 \text{ feet}^3$$

(#22) since garlic varies w/ distance from center, every slice has equally $\frac{1}{6}$ of the total garlic

radius = 7

$$\int_0^7 g(x)(2\pi x)\,dx = \text{total garlic}$$

$$= \int_0^7 \frac{x}{(x^3+2)^2}(2\pi x)dx = 2\pi\int_0^7 \frac{x^2\,dx}{(x^3+2)^2} \quad \begin{matrix} u = x^3+2 \\ du = 3x^2dx \end{matrix}$$

$$= \frac{2\pi}{3}\int_{x=0}^{x=7} \frac{du}{u^2} = \frac{2\pi}{3}\left[-\frac{1}{u}\right]_{x=0}^{x=7} = \frac{-2\pi}{3}\left[\frac{1}{x^3+2}\right]_0^7$$

$$= -\frac{2\pi}{3}\left[\frac{1}{345} - \frac{1}{2}\right] = 1.04 \text{ ounces total garlic}$$

each slice has $\frac{1.04}{6} = 0.173$ ounces garlic