Assignment #30 4/30/2001

$\boxed{31.1}$

#1 a) $\frac{dM}{dt} = 0.04M$

b) $\frac{dM}{dt} = 0.04M + 1000$

$\boxed{31.2}$

#16 a) $\frac{dM}{dt} = 0.04M - 8000$

b) the sign of the change in money is reflected in the sign of $\frac{dM}{dt}$. Thus, if $0.04M < 8000$, $\frac{dM}{dt}$ is negative, and the amount of money will keep decreasing, etc.

c) if the amount is constant, $\frac{dM}{dt} = 0$.

Thus, $0.04M = 8000$

M (initial deposit) $= \boxed{\$200{,}000}$

if $M_0 > \$200{,}000$, the amount will grow.

d) $M(t) = M_0 e^{0.04t} - 8000t$

we want $\frac{dM}{dt}$

$$\frac{dM}{dt} = 0.04(M_0)e^{0.04t} - 8000$$

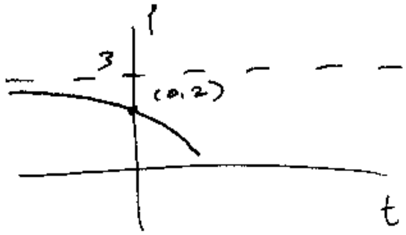
this does NOT equal $0.04M - 8000$.

(#18) a) i. $P(t) = Ce^{kt} + \frac{E}{k}$ we know $P(0) = 2$

$P(0) = 2 = Ce^0 + \frac{E}{k} = C + \frac{E}{k} \Rightarrow C = 2 - \frac{E}{k}$

$$\boxed{P = \left(2 - \frac{E}{k}\right)e^{kt} + \frac{E}{k}}$$

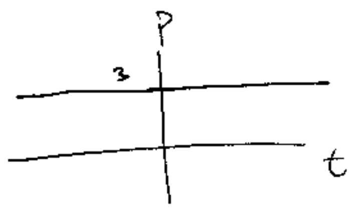
ii



b) i $3 = C + \frac{E}{k} \Rightarrow C = 3 - \frac{E}{k}$

$$\boxed{P = \left(3 - \frac{E}{k}\right)e^{kt} + \frac{E}{k}}$$

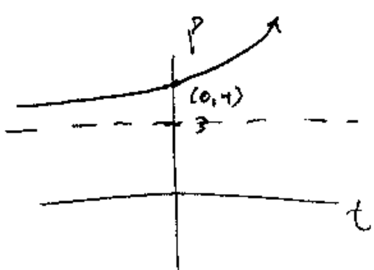
ii



c) i. $4 = C + \frac{E}{k}$ $C = 4 - \frac{E}{k}$

$$\boxed{P = \left(4 - \frac{E}{k}\right)e^{kt} + \frac{E}{k}}$$

ii



Additional Handout Problems

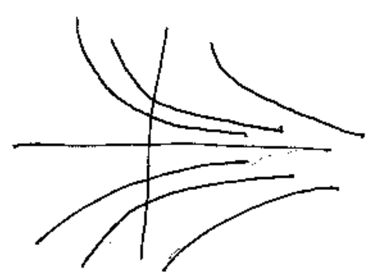
(1) a) $\frac{dy}{dt} = -y$

i. if $\frac{dy}{dt} = 0$, $\underline{y = 0}$

ii. $y < 0$

iii. $y > 0$

iv. $y(t) = Ce^{-t}$



v. $y(0) = 1$ $1 = C$

$$\boxed{y(t) = e^{-t}}$$

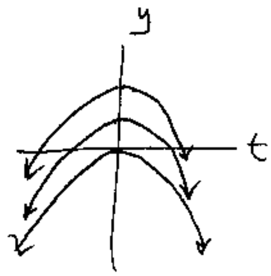
b) $\frac{dy}{dt} = -t$

i. $t=0$

ii. $t<0$

iii. $t>0$

iv. $y(t) = -\frac{1}{2}t^2 + C$



v. $y(0)=1$

$1 = 0 + C \Rightarrow C=1$

$$\boxed{y(t) = -\frac{t^2}{2} + 1}$$

c) $\frac{dy}{dt} = y-2$

i. $y=2$

ii. $y>2$

iii. $y<2$

iv. $\frac{dy}{dt} = y-2$ let $P = y-2$

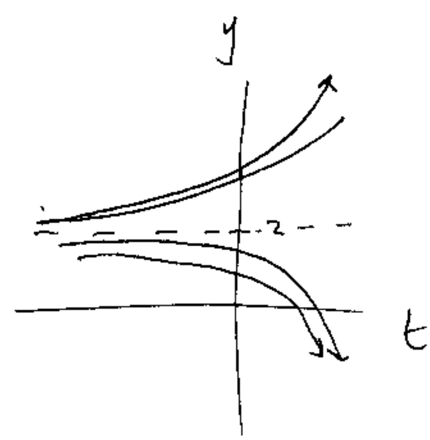
$\frac{dP}{dt} = \frac{dy}{dt}$

$\frac{dy}{dt} = \frac{dP}{dt} = P$

solution: $P = Ce^t$

$y-2 = Ce^t$

$$\boxed{y = Ce^t + 2}$$



v. $y(0)=1$

$1 = C+2$

$-1 = C$

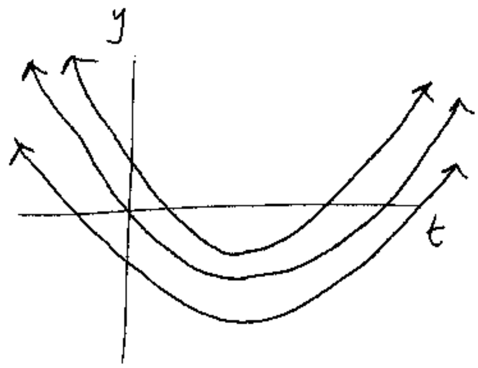
$$\boxed{y(t) = -e^t + 2}$$

d) $\frac{dy}{dt} = t - 2$

i. $t = 2$

ii. $t > 2$

iii. $t < 2$



iv. $\frac{dy}{dt} = t - 2$

$$\boxed{y(t) = \frac{t^2}{2} - 2t + C}$$

v. $y(0) = 1$

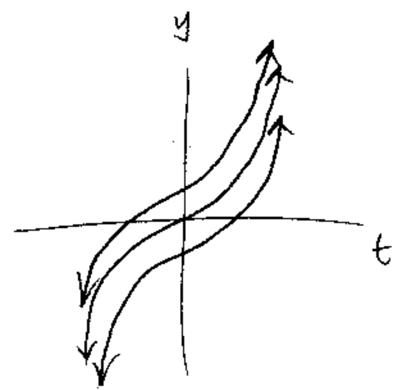
$1 = C$ $\quad \boxed{y(t) = \frac{t^2}{2} - 2t + 1}$

e) $\frac{dy}{dt} = t^2$

i. $t = 0$

ii. ~~t~~ all t

iii. ~~no t~~ $\emptyset$



iv. $\frac{dy}{dt} = t^2$

$$\boxed{y(t) = \frac{t^3}{3} + C}$$

v. $y(0) = 1$

$1 = C$ $\quad \boxed{y(t) = \frac{t^3}{3} + 1}$

#2) a) VII ; $\frac{dy}{dt} > 0$ in top left, where $2y > t$

$\frac{dy}{dt} < 0$ in bottom right, where $2y - t$ negative

b) II ; we know $\frac{dy}{dt} = 0$ @ $y = \pm 1$

c) I ; $\frac{dy}{dt} < 0$ @ $y > 1$

$\frac{dy}{dt} > 0$ @ $y < 1$

d) V ; slope decreases to zero as you move right OR left

e) IV ; ~~[illegible]~~

$\frac{dy}{dt} \to \infty$ @ $t = 0$

f) VI ; $\frac{dy}{dt} \to \infty$ @ $y = 0$

g) III ; $\frac{dy}{dt} < 0$ in 1st quadrant + 3rd quadrant

$\frac{dy}{dt} > 0$ in 2nd and 4th quadrants