Assignment #31

5/02/01

31.3

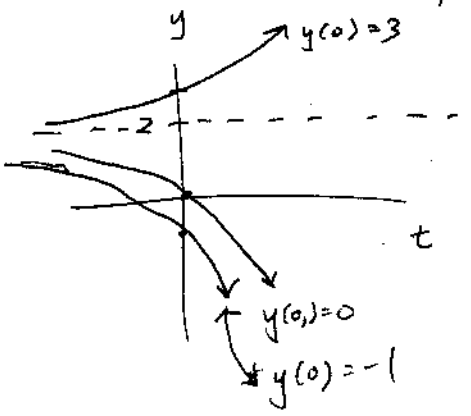
#1

a) $\frac{dy}{dt} = 4y - 8$

when $y > 2$, $\frac{dy}{dt} > 0$

when $y < 2$, $\frac{dy}{dt} < 0$

when $y = 2$, $\frac{dy}{dt} = 0$

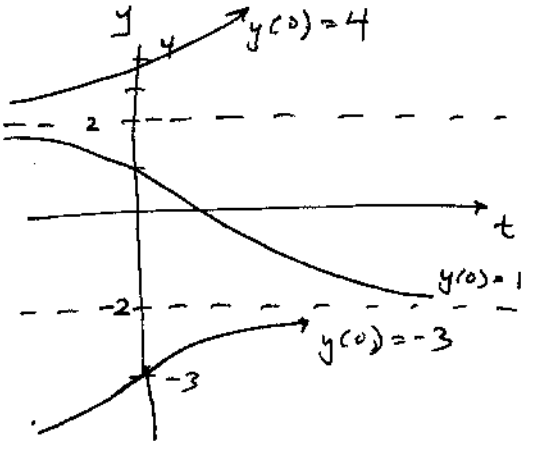


b) $\frac{dy}{dt} = y^2 - 4$

$\frac{dy}{dt} = 0$ @ $y = \{2, -2\}$

$\frac{dy}{dt} < 0$ @ $-2 < y < 2$

$\frac{dy}{dt} > 0$ @ $y < -2$ and $y > 2$



c) $\frac{dy}{dt} = (y-1)(y-2)(y+1)$

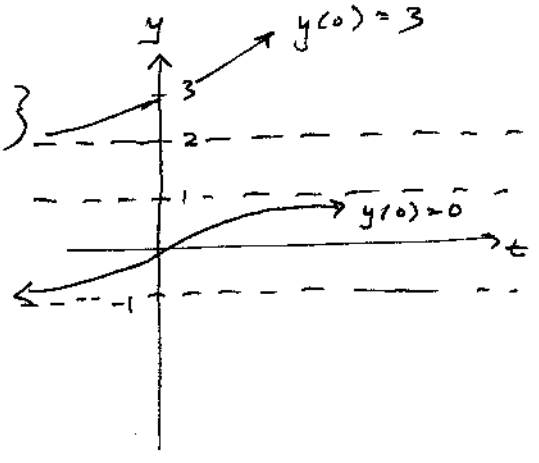
$\frac{dy}{dt} = 0$ @ $y = \{1, 2, -1\}$

when: $y > 2$, $\frac{dy}{dt} > 0$

$1 < y < 2$, $\frac{dy}{dt} < 0$

$-1 < y < 1$, $\frac{dy}{dt} > 0$

$y < -1$, $\frac{dy}{dt} < 0$



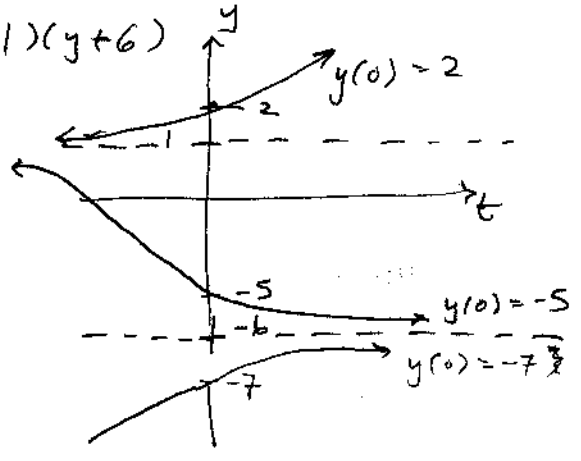
d) $\frac{dy}{dt} = y^2 + 5y - 6 = (y-1)(y+6)$

$\frac{dy}{dt} = 0$ @ $y = \{1, -6\}$

when $y > 1$, $\frac{dy}{dt} > 0$

$-6 < y < 1$, $\frac{dy}{dt} < 0$

$y < -6$, $\frac{dy}{dt} > 0$



(#2) a) $\frac{dy}{dt} = 0$ @ $y = 3$

when $y > 3$, $\frac{dy}{dt} < 0$

when $y < 3$, $\frac{dy}{dt} > 0$

$\boxed{\frac{dy}{dt} = 3 - y}$

b) $\frac{dy}{dt} = 0$ @ $y = 3$

when $y > 3$, $\frac{dy}{dt} > 0$

when $y < 3$, $\frac{dy}{dt} < 0$

$\boxed{\frac{dy}{dt} = y - 3}$

c) $\frac{dy}{dt} = 0$ @ $y = \{0, 2\}$

when $y > 2$, $\frac{dy}{dt} < 0$

$0 < y < 2$, $\frac{dy}{dt} > 0$

$y < 0$, $\frac{dy}{dt} < 0$

$\boxed{\frac{dy}{dt} = -y(y-2)}$

d) $\frac{dy}{dt} = 0$ @ $y = \{2, -2\}$

$y > 2$, $\frac{dy}{dt} > 0$

$-2 < y < 2$, $\frac{dy}{dt} < 0$

$y < -2$, $\frac{dy}{dt} > 0$

$\boxed{\frac{dy}{dt} = (y-2)(y+2)}$

(#3) a) equilibria @ $y = 3$. Stable

b) equilibria @ $y = 3$. Unstable

c) equilibria @ $y = 0, 2$.

$y = 0$ Unstable

$y = 2$ Stable

d) equilibria: $y = 2$ Unstable

$y = -2$ Stable

(#4) a) $\frac{dP}{dt} = kP(L-P)$ where k & L are constant

$\frac{dP}{dt} = 0$ @ $P = \{0, L\}$

b) $P(t) = \frac{L}{1+Ce^{-kLt}}$

$$\frac{dP}{dt} = L\frac{d}{dt}(1+Ce^{-kLt})^{-1} = -L(1+Ce^{-kLt})^{-2}(-kLCe^{-kLt})$$

$$= \boxed{k\underbrace{\frac{L}{1+Ce^{-kLt}}}_{P} \cdot \underbrace{\frac{CLe^{-kLt}}{1+Ce^{-kLt}}}_{(L-P)}}$$

$$\Downarrow$$

$$L-P = L - \frac{L}{1+Ce^{-kLt}} = \frac{CLe^{-kLt}}{1+Ce^{-kLt}}$$

(#5) a) $P(0) = 100 = 0.100$ Thousands

* units of numbers are in thousands.

"support" means $\frac{dP}{dt} = 0$. No change.

$N = 0.02P$ $\quad P(0) = 100$

$N = (0.02)(100) = 2$ Thousands

b) $N = 1$; $\frac{dP}{dt} = 0$

$0.02P = 1$

$P = 50$ thousand.

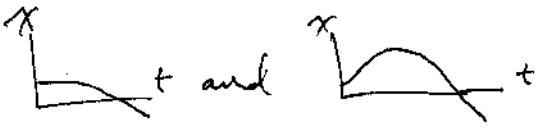
(#6) (d.) $\frac{dy}{dt} = y^2 - 1$

so, we know $\frac{dy}{dt} < 0$ in the region $-1 < y < 1$.

This is only true of graph (d.)

(#7) $\frac{dx}{dt} = x^2(2-x)$

so, $\frac{dx}{dt} = 0$ @ $x = 0, 2$



eliminate

because $\frac{dx}{dt} \neq 0$ @ $x = 0$.

when $0 < x < 2$, $\frac{dx}{dt} > 0$.

This fits