Assignment #32

5/4/2001

31.3

#8 a) If it is a solution, it should "fit" the differential equation. Ie. it's derivative should equal the expression for $\frac{dy}{dt}$.

b) A general solution is a family of many solutions that all "fit" the differential equation. A particular solution is a specific one of that family. The difference could be a "+C" at the end, OR, it could have a constant C multiplied by part (or all) of the function.

#9 a) where $\frac{dP}{dt}=0$.

$$0.01P = 0.0025P^2$$

$$\boxed{4 \text{ crocodiles} = P}$$

b) $$\frac{dP}{dt} = 0.01P - 0.0025P^2$$

$$\frac{d^2P}{dt^2} = \frac{d}{dt}\left(0.01P - 0.0025P^2\right)$$

$$= 0.01\frac{dP}{dt} - 2(0.0025)P\frac{dP}{dt}$$

$$= 0.01\left(0.01P - 0.0025P^2\right) - (0.005)P(0.01P - 0.0025P^2)$$

$$= 0.0001P - 0.000075P^2 + 0.0000125P^3$$

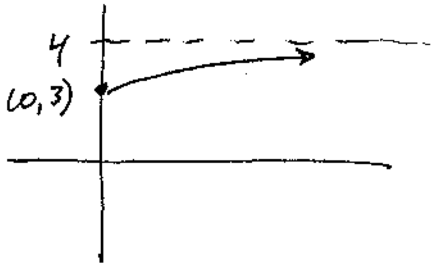
c) $\frac{d^3P}{dt^2} = 0$

$0 = 0.0001P - 0.000075P^2 + 0.0000125P^3$

$P = \{0, 4, 2\}$ $\rightarrow$ max

$\boxed{P = 2}$

d)



(#11) a) $\frac{dy}{dt} = 2y - 6$ let $P = y - 3$

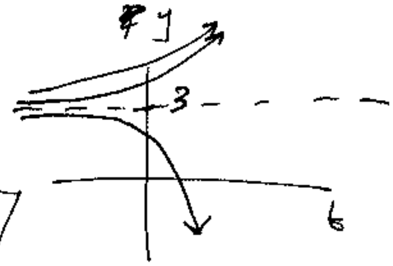
$\frac{dP}{dt} = \frac{dy}{dt} = \cancel{2a}\ 2P$

$\frac{dP}{dt} = 2P$

solution: $P = Ce^{2t}$

$y - 3 = Ce^{2t}$

$\boxed{y = Ce^{2t} + 3}$

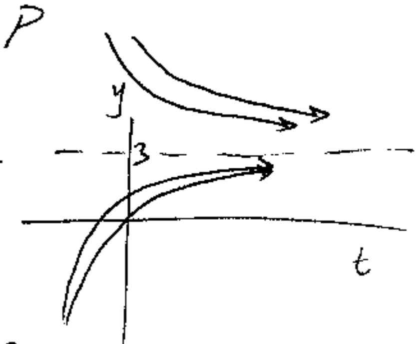


b) $\frac{dy}{dt} = 6 - 2y$ let $P = 3 - y$

$\frac{dP}{dt} = -\frac{dy}{dt} = -2P$

$\frac{dP}{dt} = -2P$

solution: $P = Ce^{-2t}$

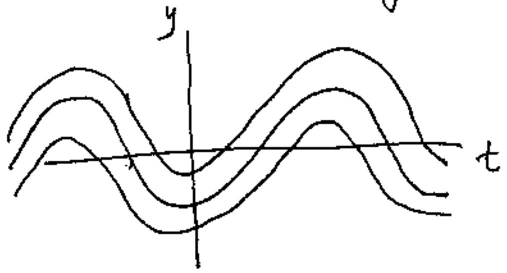


$3 - y = Ce^{-2t}$

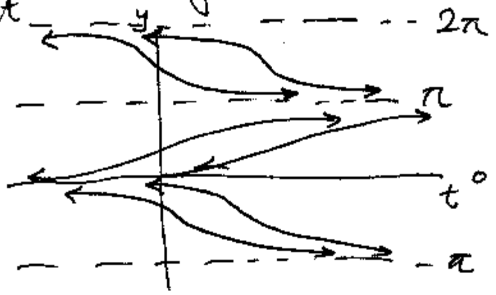
$-y = Ce^{-2t} - 3$

$\boxed{y = 3 - Ce^{-2t}}$

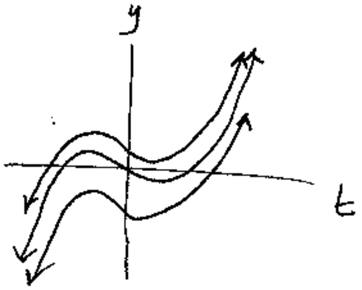
(#12) a) $\frac{dy}{dt} = \sin t \Rightarrow y = -\cos t + c$



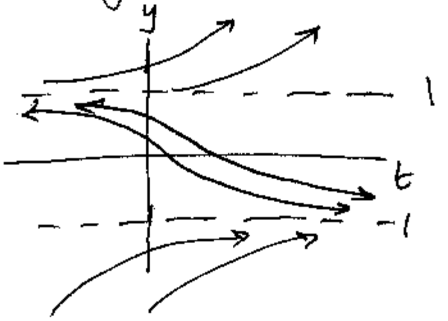
b) $\frac{dy}{dt} = \sin y$



(#15) a) $\frac{dy}{dt} = t^2 - 1 \Rightarrow y = \frac{t^3}{3} - t + c$



b) $\frac{dy}{dt} = y^2 - 1 \Rightarrow \frac{dy}{dt} = (y+1)(y-1)$

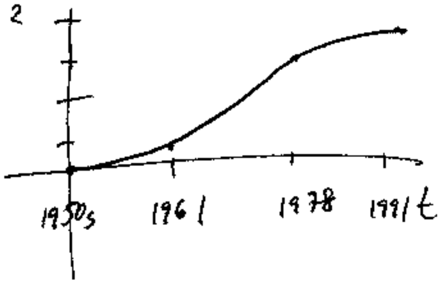


(#17) a) $\frac{dy}{dt} = (y+1)(y-4)$

b) $\frac{dy}{dt} = -(y+1)(y-4)$

c) $\frac{dy}{dt} = (y+1)^2(y-4)$

(#21)



Logistic model is better because it applies to a situation in which growth is slowed and limited when the population reaches a certain level.

Additional Problems

a) $\frac{dy}{dt} = -t/y$

$\int y\,dy = \int -t\,dt$

$\frac{y^2}{2} = -\frac{t^2}{2} + C$

$\boxed{y = \sqrt{-t^2 + C}}$

b) $\frac{dy}{dt} = -y$

$\boxed{y = Ce^{-t}}$

c) $\frac{dy}{dt} = y^2$

$\int \frac{dy}{y^2} = \int dt$

$-y^{-1} = t + C$

$y^{-1} = -t + C$

$\boxed{y = \frac{1}{-t+C}}$

d) $\frac{dy}{dt} = t^2$

$\boxed{y = \frac{1}{3}t^3 + C}$