Assignment #8 2/23/2001

$\boxed{18.2}$

#12 a) $a=3$ ; $r=3$ (finite series)

$$S = \frac{3-3^{21}}{1-3} = \frac{3-3^{21}}{-2}$$

b) $a=\frac{2}{3}$ $r=\frac{2}{3}$ (infinite series)

$$S = \frac{a}{1-r} = \frac{\frac{2}{3}}{1-\left(\frac{2}{3}\right)} = \frac{\frac{2}{3}}{\frac{1}{3}} = 2$$

c) $a = (0.2)(10) = 2$ ; $r = 10$ (infinite)

$S = \infty$ because $r > 1$

d) $a=3$ $r=0.8$ (infinite)

$$S = \frac{a}{1-r} = \frac{3}{1-(0.8)} = 15$$

e) $a=0.2$ ; $r=1.3$ (infinite)

$S = \infty$ b/c $r > 1$

f) $a=1$ ; $r = x^2$ (infinite)

$$S = \frac{a}{1-r} = \frac{1}{1-x^2}$$

#13 d) d) $a=1$ ; $r=-x$

since $|r|<1$, converges to

$$S = \frac{1}{1-(-x)} = \boxed{\frac{1}{1+x}}$$

$\boxed{18.4}$

#3 $\displaystyle\sum_{k=2}^{100} k^{k+1}$ #5 $\displaystyle\sum_{k=0}^{\infty} 10^k(-1)^k$

#8 a) $\displaystyle\sum_{k=0}^{\infty}\left(\frac{1}{2^k}\right)(-1)^k$

b) $\displaystyle\sum_{k=2}^{\infty}\left(\frac{1}{2^k}\right)(-1)^k$

12 i) $-\frac{3}{8}+\frac{3}{16}$

ii) $$r=\frac{\frac{(-1)^{n+1}\cdot\cancel{3}}{2^{n+1}}}{\frac{(-1)^n\cdot\cancel{3}}{2^n}}=\frac{(-1)}{2}=-\frac{1}{2}$$

converges b/c $|r|<1$.

iii) $$\text{Sum}=\frac{a}{1-r}=\frac{-\frac{3}{8}}{1-\left(-\frac{1}{2}\right)}=\frac{-\frac{\cancel{3}}{8}}{\frac{\cancel{3}}{2}}=\boxed{\frac{-1}{4}}$$

13 i) $\frac{9}{4}+\frac{27}{16}$

ii) $$r=\frac{\frac{3^{n+1}}{4^n}}{\frac{3^n}{4^{n-1}}}=\frac{3^{n+1}\cdot 4^{n-1}}{3^n\cdot 4^n}=\frac{3}{4}$$ converges

iii) $$S=\frac{a}{1-r}=\frac{\frac{9}{4}}{1-\frac{3}{4}}=\frac{\frac{9}{4}}{\frac{1}{4}}=\boxed{9}$$

$\boxed{18.5}$

(#9) the Present Value of all the 180 payments, combined, equals the initial amount borrowed!

$$\$1000 = (\text{Present Value})\left(1+\frac{r}{n}\right)^{nt} = (PV)\left(1+\frac{0.06}{12}\right)^{12t}$$

$$PV = \frac{\$1000}{(1.005)^{12t}}$$

$$\text{Initial Borrow} = \frac{\$1000}{(1.005)^{12(\frac{1}{12})}} + \frac{\$1000}{(1.005)^{12(\frac{2}{12})}} + \dots + \frac{\$1000}{(1.005)^{12(\frac{180}{12})}}$$

$$= \frac{\$1000}{(1.005)^1} + \frac{\$1000}{(1.005)^2} + \dots + \frac{\$1000}{(1.005)^{180}}$$

finite geometric series!

$$a = \frac{\$1000}{1.005} \qquad r = \frac{\frac{\$1000}{(1.005)^2}}{\frac{1000}{1.005}} = \frac{1}{(1.005)}$$

$$S = \frac{a - a_{181}}{1-r} = \frac{\frac{\$1000}{1.005} - \frac{\$1000}{(1.005)^{181}}}{1-\frac{1}{1.005}}$$

$$= \boxed{\$118,503.51}$$

(#11) the FUTURE value of deposits, combined, equals the eventual \$4000.

$$\text{Future Value} = \underset{\text{monthly deposit}}{M_0}\left(1+\frac{0.045}{12}\right)^{12t}$$

$$= M_0(1.00375)^{12t}$$

Cumulative Future Values $= \$4000$

$$= M_0(1.00375)^{12\left(\frac{23}{12}\right)} + M_0(1.00375)^{12\left(\frac{22}{12}\right)} + \dots + M_0(1.00375)^{12\left(\frac{1}{12}\right)} + M_0$$

finite geometric series

$$M_0 + M_0(1.00375)^1 + \dots + M_0(1.00375)^{23}$$

$$a = M_0 \qquad r = 1.00375$$

$$S = \frac{a - ar^{24}}{1-r} = \frac{M_0(1 - 1.00375^{24})}{1 - 1.00375}$$

$$4000 = \frac{M_0(1 - 1.00375^{24})}{(1 - 1.00375)}$$

$$-15 = M_0(-0.094)$$

$$\boxed{\$159.59 = M_0}$$