Problem Set #7

Pick up a copy of the Coursebook at Gnomen Copy, next to Toscannini's.

Read 18.1 and 18.5 from page 579 through the top of page 589.
Do: 18.1 # 21, 29, 31, 33, 18.5 # 1, 2, 13

Also do the following two problems. Each asks you to model a situation using a differential equation.

1. Throughout history, various religious groups have sprung up, only to disappear. One such group was an early Christian sect known as the Gnostics. The Gnostics had their own special gospels, besides the now canonical Matthew, Mark, Luke and John. Some early religious institutions were not impressed by the Gnostics or their special gospels, and worked very hard to suppress these writings.

   In the first century of the common era, a group of Gnostics was struggling to survive. Their strategy was to make lots of copies of their special gospels. Because the printing press had not been invented, these copies were made by hand The authorities responded by trying to seize and destroy as many copies as possible.

   Suppose that the authorities succeed in destroying 75 copies of the Gnostic gospels per month, and that the Gnostics are able to produce gospels at a rate proportional to the number of gospels that they currently have. The constant of proportionality is 0.15.

   (a) In terms of the way that copies of the special gospels are made, explain why it is reasonable to assume that the rate at which gospels are produced is proportional to the number of gospels that the Gnostics currently have.

   (b) Write a differential equation whose solution is $G(t)$, the number of gospels that the Gnostics have at time t (in months).

   (c) Using the initial condition, $G(0) = G_0$, solve the differential equation from part (b). Your answer may contain the constant G_0.

   (d) If the sect survives only as long as they have copies of their special gospels, how many copies do the Gnostics need initially to guarantee that their sect will survive?

2. A cheetah is stalking its prey. Cheetahs are known for their phenomenal speed over short time periods. Top speeds range from 90 to 112 kilometers per hour. (Source: www.dewildt.org.za/cheetah)

   Let $x(t)$ denote the position of the cheetah at time t, where t is measured in hours. Let $t = 0$ denote the time at which the cheetah begins accelerating from its stalking speed of 2 kilometers per hour. At $t = \frac{1}{120}$ the cheetah is moving at a speed of 42 kilometers per hour. Suppose that the cheetah's velocity is a linear function of t.

   (a) Write a differential equation whose solution is $x(t)$, the cheetah's position at time t.

   (b) How long does it take for the cheetah to reach a speed of 100 kilometers per hour?

   (c) Solve this differential equation to find the position of the cheetah at time t.

   (d) How far has the cheetah travelled from time $t = 0$ to the time when its velocity reached 100 km/hr?