b) Method 1: solve for y explicitly

$$4(x-1)^2 + 9(y-3)^2 = 36$$

$$9(y-3)^2 = 36 - 4(x-1)^2$$

$$(y-3)^2 = \frac{36 - 4(x-1)^2}{9}$$

$$y = \sqrt{\frac{36 - 4(x-1)^2}{9}} + 3$$

$$\frac{dy}{dx} = \frac{d}{dx}\sqrt{\frac{36 - 4(x-1)^2}{9}} + 0 = \frac{d}{dx}\left(\frac{36 - 4(x-1)^2}{9}\right)^{1/2}$$

$$\frac{dy}{dx} = \frac{1}{2}\left(\frac{36 - 4(x-1)^2}{9}\right)^{-1/2} \cdot \frac{d}{dx}\left(\frac{36}{9} - \frac{4}{9}(x-1)^2\right)$$

$$= \frac{1}{2}\left(\frac{36 - 4(x-1)^2}{9}\right)^{-1/2} \cdot \left(-\frac{8}{9}(x-1)\right)$$

fit $x=1$ and $y=5$ for slope

Method 2: implicit differentiation

$$4(x-1)^2 + 9(y-3)^2 = 36$$

$$4(x^2 - 2x + 1) + 9(y^2 - 6y + 9) = 36$$

$$4x^2 - 8x + 4 + 9y^2 - 54y + 81 = 36$$

$$4x^2 - 8x + 9y^2 - 54y = -49$$

(differentiate)
$$\frac{d}{dx}(4x^2 - 8x + 9y^2 - 54y) = \frac{d}{dx}(-49)$$

$$8x - 8 + 18y\frac{dy}{dx} - 54\frac{dy}{dx} = 0$$

$$\frac{dy}{dx}(18y - 54) = -8x + 8$$

$$\frac{dy}{dx} = \frac{-8x + 8}{18y - 54}$$

@ $(1,5)$: $\frac{dy}{dx} = \frac{-8 + 8}{18(5) - 54} = 0$