

(#9) a) $x^2 + y^2 = 4$

$$\frac{d}{dx}(x^2 + y^2) = \frac{d}{dx} 4$$

$$2x + 2y \frac{dy}{dx} = 0$$

$$2y \frac{dy}{dx} = -2x$$

$$\frac{dy}{dx} = \frac{-2x}{2y} = -\frac{x}{y} \quad \checkmark$$

b) $\frac{d^2y}{dx^2} = \frac{d}{dx} \left(-\frac{x}{y} \right) = -\frac{d}{dx} \left(\frac{x}{y} \right)$

- use quotient rule

$$\frac{d^2y}{dx^2} = -\frac{y - x \frac{dy}{dx}}{y^2}$$

$$= -\frac{y - x \left(-\frac{x}{y} \right)}{y^2}$$

$$= -\frac{y + \frac{x^2}{y}}{y^2}$$

$$= -\frac{\frac{y^2 + x^2}{y}}{y^2}$$

$$= -\frac{y^2 + x^2}{y^3} \Rightarrow -\frac{4}{y^3} \quad \checkmark$$

(because $x^2 + y^2 = 4$)