

### Homework Assignment 11: Due at the beginning of class 3/13/02

In Questions 1, 2 and 3 of this homework assignment, you will be studying a function  $f(x)$ . The first way that you will study the function will be to use the ideas (first and second derivative, increasing/decreasing, concavity, critical points, inflection points, etc.) that you learned in Math Xa to reconstruct the graph of  $y = f(x)$  based on the graphs of  $y = f'(x)$  and  $y = f''(x)$ . You will then use an equation for  $f'(x)$  and the concept of antidifferentiation to create an equation for  $f(x)$ . In Question 3 you will verify that the graph and equation that you construct are consistent with one another.

1. In this problem, all that you can assume about the function  $f(x)$  is that:

- $f(0) = 1$ .
- The graphs of  $y = f'(x)$  and  $y = f''(x)$  are given by Figures 1 and 2 (respectively).

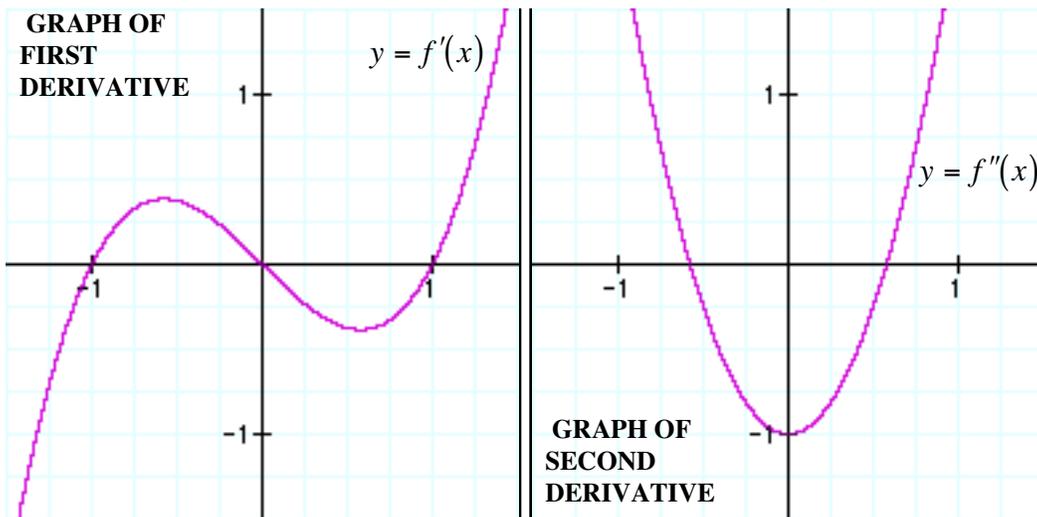
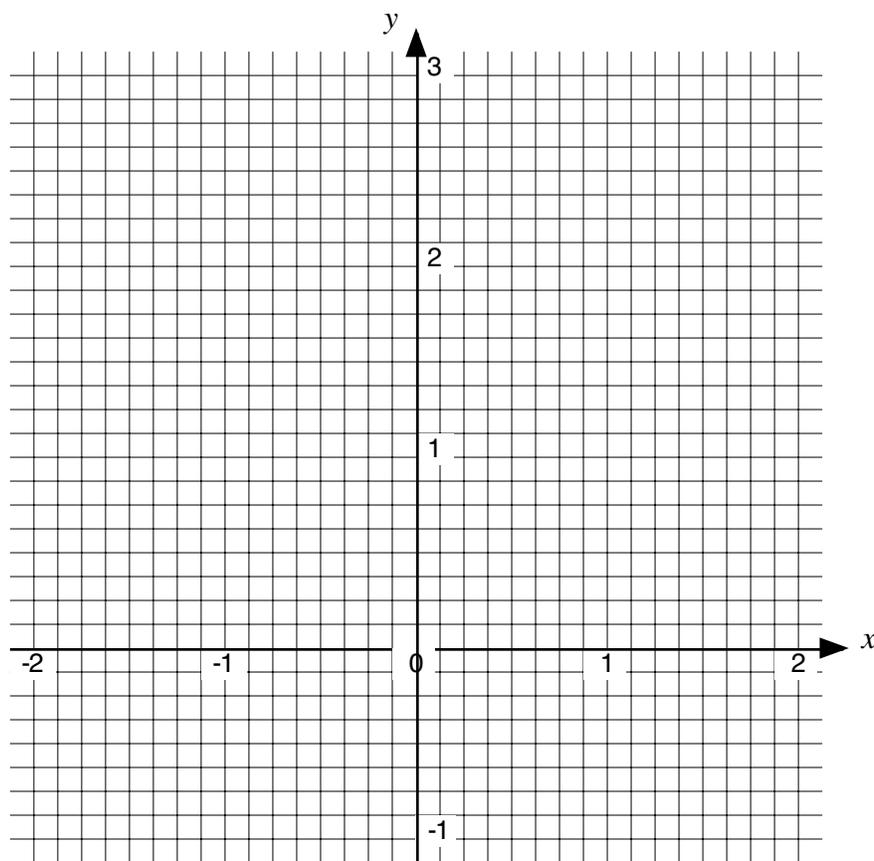


Figure 1: Graph of first derivative.

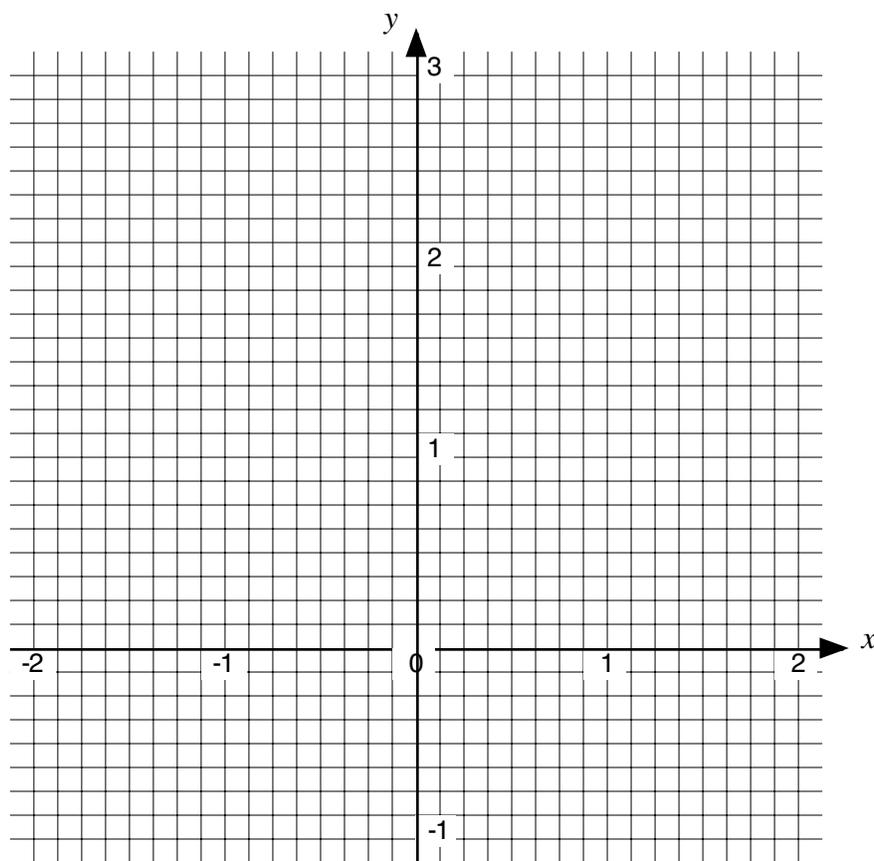
Figure 2: Graph of second derivative.

Use Figures 1 and 2 to locate the  $x$ -coordinates of all of the critical points and inflection points of  $f(x)$ . Use the axes provided below to sketch a graph of  $y = f(x)$  that is consistent with Figures 1 and 2 and the given value of the function:  $f(0) = 1$ .



**NOTE:** You should hand in a list of the  $x$ -coordinates of all critical and inflection points, together with the classifications (local maximum, local minimum) of all critical points. You should also hand in the graph of  $y = f(x)$  that you draw.

2. Use the techniques that you learned in Math Xa for finding equations for polynomial functions and Figure 1 to find an equation for the first derivative,  $f'(x)$ . (You can assume that the constant of proportionality in the polynomial is  $k = 1$ .) Use this equation to determine what  $f(x)$  must have been in order to be able to differentiate  $f(x)$  and end up with  $f'(x)$ . Be sure that you use the fact that  $f(0) = 1$  to evaluate the constant in your formula for  $f(x)$ .
  
3. Use the axes provided below to sketch the graph of  $y = f(x)$  that you found in Question 2. Comment on any similarities and differences between this graph and the graph that you drew in Question 1. (There should be some differences, for example the  $y$ -coordinates of important points might be different.)



**NOTE:** You should hand in your graph together with a list of the differences between the graphs from Question 1 and Question 3, and the reasons for these differences.

In Questions 4 and 5 you will be asked to calculate the area under some curves. Our expectation for how you will do this is the following:

- Use the graph provided to determine what sort of function would do a reasonable job of representing the trends and patterns that are present in the data.
- Use the data points and your graphing calculator to find an equation for the function.
- Create an antiderivative for the function that your graphing calculator produces.
- Use the antiderivative to evaluate the amount of area under the curve between 0 and 2 hours.

**If you are under a lot of time-pressure this week, skip to the last two pages of the assignment.**

In Questions 4 and 5 you will calculate the *glycemic index* (GI) of two different sources of carbohydrates (glucose and bread). The *glycemic index* is used by some sports nutritionists to rank complex carbohydrate-rich foods according to how the body

responds to intake of each different food. The glycemic index was introduced by David Jenkins (a medical researcher at the University of Toronto) as a physiological measure of how fast (and to what extent) different foods affect blood sugar levels. Dr. Jenkins and his colleagues were particularly interested in the use of the glycemic index in understanding the response of diabetics to different types of carbohydrate-rich foods<sup>1</sup>

Complex carbohydrates (or polysaccharides) are molecules composed of sugar molecules (disaccharides) linked together<sup>2</sup>. Polysaccharide molecules normally have between 3 and 3000 disaccharides linked together (see Figure 3<sup>3</sup> for examples). In the past<sup>4</sup> the accepted wisdom in sports nutrition (and among many scientists interested in nutrition) was that small polysaccharide molecules raised blood glucose levels quickly, but only for short periods of time, whereas larger polysaccharide molecules produced lower but much more stable levels of blood glucose for much longer periods of time.

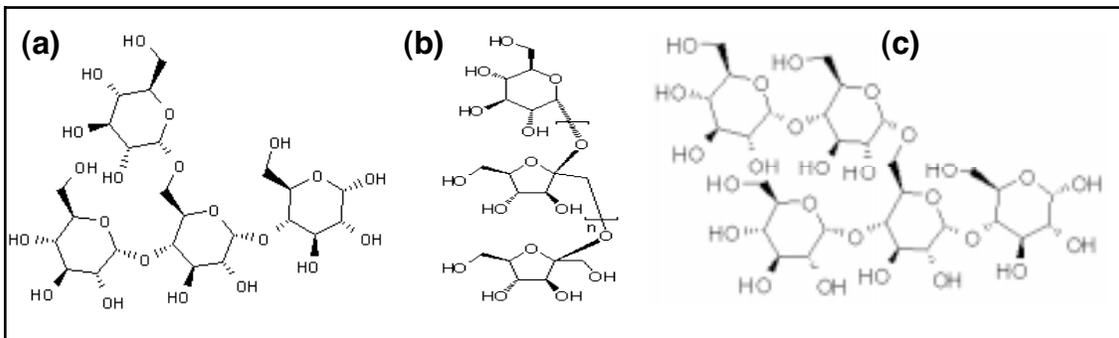


Figure 3: Three examples of polysaccharide molecules. (a) Glycogen. (b) Insulin. (c) Amylopectin.

This simplistic view of how the body responds to the ingestion of complex carbohydrates is no longer in favor among sports nutritionists. Nowadays, a popular practice among many nutritionists is to use the glycemic indices of foods as indicators of when athletes can best consume different types of food in to maximize their performance. Many registered sports dieticians now suggest that athletes:

- Consume foods that have a low glycemic index before exercise to promote sustained carbohydrate availability while exercising,
- Consume food and drinks with a high glycemic index while exercising to promote glucose oxidation during exercise (the fundamental source of energy for muscles).
- Consume foods with a high glycemic index after exercising to promote glycogen<sup>5</sup> replenishment after exercise.

<sup>1</sup> See, for example: D.J.A. Jenkins. (1981) "Glycemic index of foods: A physiological basis for carbohydrate exchange." *The American Journal of Clinical Nutrition*, **34**: 362-366.

<sup>2</sup>Source: R.C. Atkins and F.A. Carey. *Organic Chemistry. A Brief Course. Third Edition.* Boston, MA: McGraw Hill, 2002.

<sup>3</sup> Image source: <http://www.genome.ad.jp/>

<sup>4</sup> Source: B. Seebhar. "The glycemic index in endurance sport." Available on-line at: <http://www.trisite.com/>

<sup>5</sup> Glycogen is a polysaccharide. Glycogen is formed from glucose in the bloodstream by a biochemical process called glycogenesis. Glycogen is mainly stored in the liver, however it may also be formed and

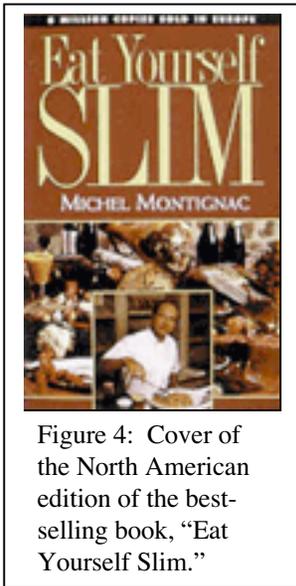


Figure 4: Cover of the North American edition of the best-selling book, “Eat Yourself Slim.”

It should be noted, however, that this advice is highly controversial. Many registered sports dietitians<sup>6</sup> openly admit that there is not enough evidence to support the use of the dietary plan outlined above for all athletes. Furthermore, some scientific research appears to contradict the suggestion that following such a dietary regimen will affect athletic performance at all<sup>7</sup>.

The glycemic index has also been suggested (somewhat controversially<sup>8</sup>) as a basis for weight-loss programs with astonishing titles<sup>9</sup> such as: “Eat Yourself Thin” and “Sugar Busters!” (see Figure 4<sup>10</sup>).

Examples of food with low, moderate and high glycemic indices are given in Table 1 (below)<sup>11</sup>.

stored in muscle tissue. During short periods of strenuous activity, energy may be released in the muscles by the direct conversion of glycogen to lactic acid, depleting the levels of glycogen in the body. The process of glycogenesis (together with the hydrolysis of glycogen to glucose) are the usual mechanism by which the body maintains normal blood sugar levels.

<sup>6</sup> Source: B. Seebohar. “The glycemic index in endurance sport.” Available on-line at: <http://www.trisite.com/>

<sup>7</sup> See, for example: S.R. Stannard, M.W. Thompson and J.C.B. Miller. (2000) “The effect of glycemic index on plasma glucose and lactate levels during incremental exercise.” *International Journal of Sports Nutrition*, **10**(1): 51-61. In this article, the authors reported that consumption of a different GI carbohydrates did not affect exercise performance or plasma lactate concentrations after exercise, however plasma glucose concentration was lower (after exercise) in the subjects who had consumed a high GI food immediately before exercising. Similar results were reported in: S.-L. Wee, C. Williams, S. Gray and J. Horabin. (1999) “Influence of high and low glycemic index meals on endurance running capacity.” *Medicine and Science in Sports and Exercise*, **31**(3): 393-399.

<sup>8</sup> See, for example: R. Ratnesar. (1997) “Against the grain: The low-carb zone diet rises from fad to fixture.” *Time*, **150**(25), December 15, 1997. and J. Stein. (1999) “The low-carb diet craze.” *Time*, **154**(18), November 1, 1999.

<sup>9</sup> Despite the fact that the titles of these books sound as though they are from the hit TV show, “The Simpsons,” there is a small library of titles in the “Eat Yourself Thin” and related “Sugar Busters!” publishing empire. These include:

- M. Montignac. *Eat Yourself Slim*. Baltimore, MD: Erica House, 1999.
- N. Moshier. *Eat Yourself Thin Like I Did!* Garrison, MN: Nancy’s Cookbooks, Inc., 2000.
- B. Downs. *Eat Yourself Thin Cookbook*. Cambret, 1970.
- H.L. Stewart, M.C. Bethea and S.S. Andrews. *Sugar Busters! Cut Sugar to Trim Fat*. New Orleans: Ballantine Books, 1998.
- H.L. Stewart, M.C. Bethea, S.S. Andrews and L.A. Balart. *Sugar Busters! Quick & Easy Cookbook*. New Orleans: Ballantine Books, 1999.
- H.L. Stewart, M.C. Bethea and S.S. Andrews. *Sugar Busters! Shoppers Guide*. New Orleans: Ballantine Books, 1999.
- S.S. Andrews, M.C. Bethea and H.L. Stewart. *Sugar Busters for Kids*. New Orleans: Ballantine Books, 2001.

<sup>10</sup> Image source: <http://www.epicurean.com/>

<sup>11</sup> The source of the glycemic indices used to create this table is the searchable database available on-line at:

Low glycemic index	Moderate glycemic index	High glycemic index
<ul style="list-style-type: none"> <li>• 2 scoops of low fat ice cream</li> <li>• Cheese tortellini, 180g</li> <li>• 1 cup pasta (cooked)</li> <li>• 1 cup unsweetened apple juice</li> <li>• 1 small can Ensure™</li> <li>• 2 slices Pizza Hut™ supreme pizza</li> <li>• 1 protein bar</li> <li>• 1 cup beans, lentils, etc.</li> <li>• 1 cup non-fat milk</li> </ul>	<ul style="list-style-type: none"> <li>• 1 bagel</li> <li>• 1 blueberry muffin</li> <li>• 1 slice whole wheat bread</li> <li>• 1 cup Kelloggs “Just Right™” cereal</li> <li>• 1 slice of melon</li> <li>• 1 teaspoon of sugar</li> <li>• 1 Pop Tart™</li> <li>• French fries, 120g</li> <li>• 1 cup Gatorade™</li> <li>• 1 cup orange juice</li> </ul>	<ul style="list-style-type: none"> <li>• 2 slices white bread</li> <li>• French bread (baguette), 30g</li> <li>• Lucozade™ energy drink, 300ml</li> <li>• Roll-ups™ chewy fruit candy</li> <li>• 1 cup glutinous white rice</li> <li>• 1 pancake from powdered mixture</li> <li>• 180g mashed potatoes</li> </ul>

Table 1: Glycemic indices for some foods.

In scientific studies of the glycemic index, test subjects are instructed to fast for 10-12 hours (i.e. overnight) and then given a measured<sup>12</sup> amount of a certain food to eat. The subjects’ blood glucose concentration is measured for a number of hours after the food was consumed, and the results plotted on a graph of blood glucose concentration versus time (see Figure 5<sup>13</sup> for an example). The glycemic index of a food is calculated by measuring the area under the graph of blood glucose concentration versus time. The area used is normally the area between the time at which the person ingested the food and 2 hours after the food was eaten.

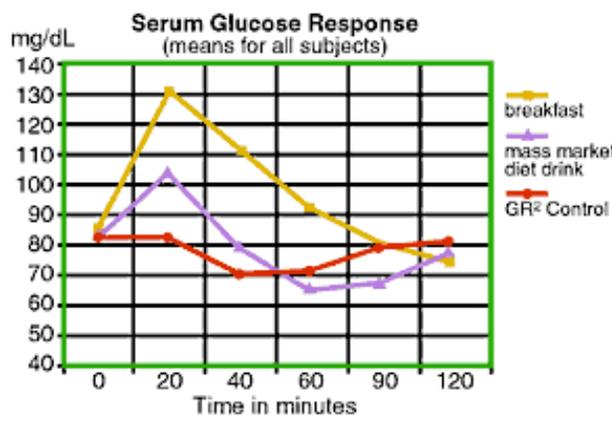


Figure 5: Example of a graph showing blood glucose concentration (the units here are milligrams of glucose per deciliter of blood) versus time.

<http://www.glycemicindex.com/> For the purposes of this table, foods with a “high glycemic index” are those with  $GI \geq 90$ , foods with a “moderate glycemic index” are those with  $50 < GI < 90$ . Foods with a “low glycemic index” are those with  $GI \leq 50$ . These numbers have all been normalized to a scale on which the glycemic index of 50g of glucose powder produces a GI of 100.

<sup>12</sup> The amount of food is usually chosen so that the food ingested will contain exactly 50g of digestible carbohydrate. The digestible part is important, as dietary fiber (typically composed of carbohydrates) is indigestible and will not affect blood glucose levels.

<sup>13</sup> Image source: <http://www.gnldbetterhealth.com/>

4. Figure 6<sup>14</sup> (below) shows the blood glucose concentration curve for a person who has just ingested 50g of glucose powder. The glucose blood concentrations are given in Table 3 (below). Calculate the glycemic index for 50g of glucose powder.

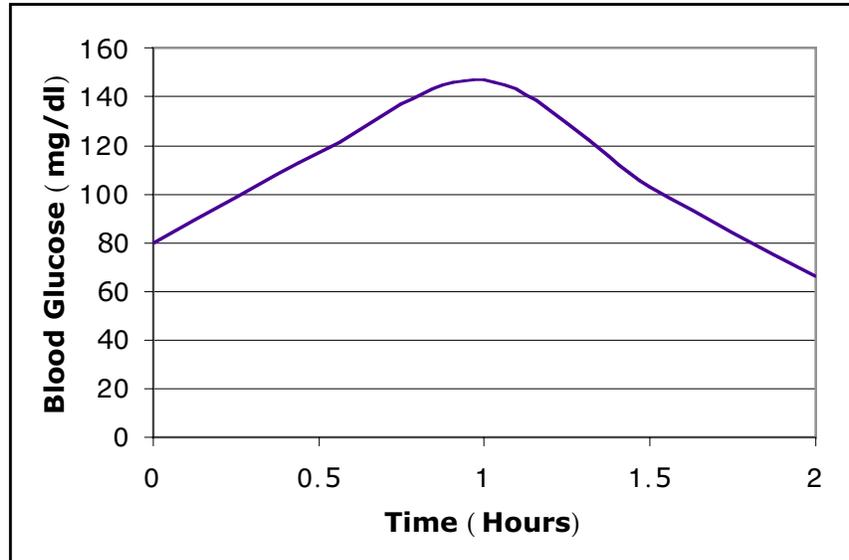


Figure 6: Serum glucose response for subject ingesting 50g of glucose powder.

Time (hours)	0	0.5	1	1.5	2
Blood glucose (mg/dl)	80	117	147	103	66

Table 2: Serum glucose levels for subject ingesting 50mg of glucose powder.

5. Figure 7 (below) shows the blood glucose concentration curve for a person who has just consumed two slices of white bread. The glucose blood concentrations are given in Table 4 (below). Calculate the glycemic index for two slices of white bread.

<sup>14</sup> The data for Figure 6 and 7 was obtained from: <http://www.prolithic.com/>

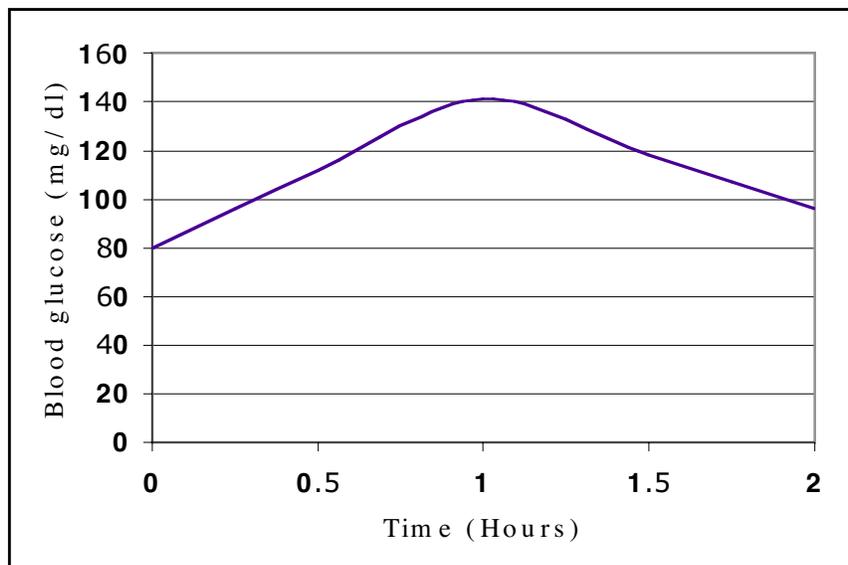


Figure 7: Serum glucose response for subject ingesting two slices of white bread.

Time (hours)	0	0.5	1	1.5	2
Blood glucose (mg/dl)	80	112	141	118	96

Table 3: Serum glucose levels for subject ingesting 2 slices of white bread.

**HINT:** Imagine that Figures 6 and 7 are derivative graphs. What you are trying to do in Questions 4 and 5 is to find the area beneath a derivative graph. As you know from the Fundamental Theorem of Calculus, the area beneath a derivative graph between  $x=a$  and  $x=b$  is equal to the change in the value of the original function between  $x=a$  and  $x=b$ . The original function in this case, is the antiderivative.

### Epilogue:

One thing that you might have wondered is what kind of food has the highest glycemic index? Of the foods that Americans normally eat, bananas, white rice (especially the “sticky” rice favored in Asian cuisine) and potatoes have the highest glycemic indices (as high as 140 or 160 according to some studies)<sup>15</sup>.

You might also have wondered what the best food is to produce a reasonable level of blood sugar over the “long haul.” According to a scientific study<sup>16</sup> the answer is pizza. In other words, according to the study cited in the footnotes below, if you want to achieve an elevated blood sugar level for an extended period of time, eat pizza.

<sup>15</sup> Source: <http://www.glycemic.com/inter.htm>

<sup>16</sup> Source: J. A. Ahern (1993) “Exaggerated hyperglycemia after a pizza meal in well-controlled diabetics.” *Diabetes Care*, **16**: 578-580.