Homework Assignment 12: Solutions

1. Based on the appearance of Figure 4 from the homework assignment (reproduced below for clarity) a cubic or a quartic polynomial could conceivably do a reasonable job of representing the patterns in the data.

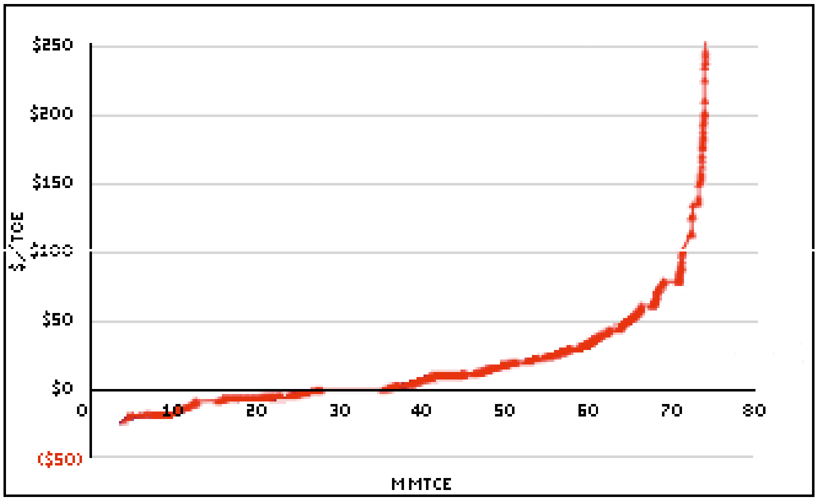

Figure 4: Marginal cost curve for removoal/reduction of the greenhouse gas, methane.

Let x represent the mass of methane reduced/removed (measured in millions of metric tons of carbon equivalent, MMTCE) and $g(x)$ represent the marginal cost (in units of dollars per metric ton carbon equivalent, $/TCE).

A degree 3 or degree 3 polynomial (cubic or quartic) would be perfectly acceptable for an answer here, and the equations obtained by using cubic or quartic regression on a TI-83 with the data given in Table 1 of the homework assignment are given below.

Type of Regression	Equation for Marginal cost
Cubic	$g(x) = 0.001389 \cdot x^3 - 0.132738 \cdot x^2 + 4.426587 \cdot x - 48.92857$
Quartic	$g(x) = 0.0000436 \cdot x^4 - 0.00558 \cdot x^3 + 0.24375 \cdot x^2 - 3.38943 \cdot x + 0.35714$

To see how each of these curves correspond to the data, the graphs of the curves and the data points from Table 1 of the homework assignment are shown in Figures 1 and 2 below.

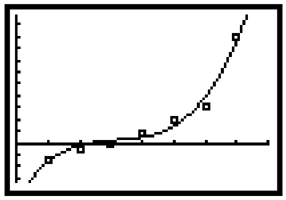

Figure 1: Data points from Table 1 of homework and graph of cubic polynomial.

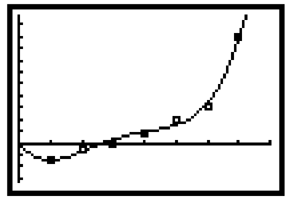

Figure 2: Data points from Table 1 of homework and graph of quartic polynomial.

2. As *marginal cost* is the derivative of *cost*, it follows that if you reverse this relationship, that *cost* should be an antiderivative of *marginal cost*. Therefore, to find an equation for the cost function, you could find an equation for the antiderivative of the polynomial that you calculated in Question 1.

Let $G(x)$ represent the cost of reducing/removing x million metric tons carbon equivalent (MMTCE) of methane. Then $G(x)$ is an antiderivative of $g(x)$, the polynomial function found in Question 1. The antiderivatives of the cubic and quartic polynomials , $g(x)$, are given in the table below. Note that the symbol "*C*' is intended to represent an unspecified constant.

Type of Regression	Antiderivative of $g(x)$
Cubic	$G(x)=\frac{0.001389}{4}\cdot x^4-\frac{0.132738}{3}\cdot x^3+\frac{4.426587}{2}\cdot x^2-48.92857\cdot x+C$
Quartic	$G(x)=\frac{0.0000436}{5}\cdot x^5-\frac{0.00558}{4}\cdot x^4+\frac{0.24375}{3}\cdot x^3-\frac{3.38943}{2}\cdot x^2+0.35714\cdot x+C$

As noted in the homework assignment, these cost functions include an unspecified constant[1], *C*. If we knew one value of the cost function, then we could use that to determine the numerical value of the constant *C*.

To deduce one value of the cost function, we can consider the case where no effort whatsoever is made to reduce emissions of CH_4 into (nor any effort made to remove the greenhouse gas CH_4 from) the atmosphere. As this would involve taking no action whatsoever and would expend no resources, it should be relatively inexpensive. Therefore, I would guess that if no effort whatsoever is made to reduce or remove methane from the atmosphere, then the cost should be zero. In symbols:

$$G(0) = 0.$$

Substituting $x = 0$ and $G(0) = 0$ into the two antiderivatives given in the table above gives that the unspecified constant is equal to zero. Therefore, the possible cost functions are:

[1] In the language of microeconomics, this unspecified constant would be the *fixed cost*.

Type of Regression	Cost Function, $G(x)$
Cubic	$G(x)=\frac{0.001389}{4}\cdot x^4-\frac{0.132738}{3}\cdot x^3+\frac{4.426587}{2}\cdot x^2-48.92857\cdot x$
Quartic	$G(x)=\frac{0.0000436}{5}\cdot x^5-\frac{0.00558}{4}\cdot x^4+\frac{0.24375}{3}\cdot x^3-\frac{3.38943}{2}\cdot x^2+0.35714\cdot x$

3. To determine the units of the area under the marginal cost curve in Figure 4 of the homework assignment, you can multiply the units of the vertical axis by the units of the horizontal axis. The resulting units will be the units of the area beneath the marginal cost curve. This calculation will look something like:

$$(Vertical\_Units)\cdot(Horizontal\_Units)=\frac{Dollars}{TCE}\cdot\frac{Millions\_TCE}{1}=\textit{Millions of Dollars}.$$

So, the units of the area under the marginal cost curve are: *Millions of dollars*. Therefore, the outputs from the cost functions calculated in Question 2 will be expressed in units of millions of dollars.

To calculate the cost of reducing/removing 30 million metric tons carbon equivalent of methane, you can simply substitute $x = 30$ into the cost function that you calculated in Question 2. The results of doing this for the cost functions based on cubic and quartic regression are shown in the table below.

Type of Regression	Cost of removing/reducing 30 MMTCE Methane	Units of cost function
Cubic	−389.26245	Millions of dollars
Quartic	−238.8333	Millions of dollars

4. To calculate the cost of removing/reducing 4256.9 MMTCE of methane, you can substitute $x = 4256.9$ into the cost function that you calculated in Question 2. The results of doing this are shown in the table given below.

Type of Regression	Cost of removing/reducing 4256.9 MMTCE Methane	Units of cost function
Cubic	1.106559494×10^{11}	Millions of dollars
Quartic	1.17375891×10^{13}	Millions of dollars

To determine the percentage of the GDP of the United States that would have to be devoted to atmospheric cleanup in order to remove the 4256.9 MMTCE of anthrogenic methane currently in the atmosphere, you can multiply the value of the cost function by 1000000 (so that it is expressed in dollars rather than millions of dollars), divide by the GDP of the United States ($9,969,000,000,000) and multiply by 100%. In symbols:

$$\textit{Percentage of GDP for Atmospheric Cleanup} = \frac{G(4256.9)\times 1000000}{9969000000000}\cdot\frac{100\%}{1}.$$

Using the values of $G(4256.9)$ from the previous table gives the results recorded in the next table.

Type of Regression	Percentage of United States GDP that would have to be spent on Atmospheric cleanup to remove all anthrogenic methane (%)
Cubic	1,110,000.496
Quartic	117,740,887.7

The sheer magnitude of these numbers is utterly shocking.

The number based on cubic regression suggests that it would take *all of the economic output of the United States for a period of about 10,000 years* to pay for the clean-up of one greenhouse gas (methane – and methane is not even the most prevalent greenhouse gas[2]).

The number based on the quartic polynomial is even more shocking. It suggests that the entire economic output of the United States *for more than a million years* would be required to finance the removal of *just* the methane that humans have released into the atmosphere over the last 12 years (between 1990 and 2002).

If these estimates have any merit whatsoever, then the message that they clearly send is that it is beyond the capacity of all humnkind to return the atmosphere to the pristine mixture of gases that existed before large scale human activities (such as industrial revolutions) began to release unprecedented quantities of man-made gases into the atmosphere. In this regard, the plans announced by President Bush and other world leaders to set more modest targets for the reduction and elimination of anthrogenic greenhouse gases makes sense.

5. The EPA projection is that if nothing is done whatsoever to reduce methane emissions or to remove methane from the atmosphere, then in 2012 about 177.25 MMTCE of methane will be released into the atmosphere as a result of human

[2] The most prevalent anthrogenic greenhouse gas (by mass) is carbon dioxide, CO_2.

activity in the United States. President Bush's plan pledges to reduce this by 18%. The amount that President Bush pledges to reduce/remove methane is therefore:

$$\textit{President Bush's Pledge} = (177.25)\cdot\frac{18\%}{100\%} = 31.905 \text{ MMTCE.}$$

To determine how much it will cost to reduce/remove this much methane, you can substitute $x = 31.905$ into the cost function that you calculated in Question 2. The results of substituting $x = 31.905$ into the cost functions based on cubic and quartic regression are shown in the table below.

Type of Regression	Cost (millions of dollars)
Cubic	−385.2554722
Quartic	−232.1399564

Therefore, President Bush's plan will actually make money (hence should be feasible).

Epilogue/Reassurance

From purely mathematical principles, it will come as no surprise that the value of the cost function is negative between $x = 0$ and $x \approx 50$. From a mathematical point of view this is because the marginal cost curve, $y = g(x)$, shown in Figure 4 of the homework assignment dips below the horizontal axis until the graph gets to $x \approx 30$. Therefore, the cost function (which is the "area" between the curve and the x-axis) will be negative as well.

Figure 3: Tube installed in a land fill site to allow escape of methane gas. Gas from land fills accounts for 37% of the anthrogenic methane released each year.

Why does this make sense from an economic point of view? In economics you can regard a negative cost as a positive revenue (and vice versa).

According to the EPA[3], the easiest way to reduce the amount of anthrogenic methane released to the atmosphere industries (for example, the natural gas industry) to simply be more careful about their handling of methane so that less will escape by accident, through leaky and faulty equipment, etc. Far from costing the natural gas industry money, these measures will actually return money to the industry as they will have more of their product (CH_4) to sell to their customers. Therefore, if industry simply takes extra care to reduce leaks and accidental releases of CH_4, they will not only cut down on the amount of greenhouse gas released into the atmosphere, but they will probably enjoy higher revenues from increased sales as well.

Similarly, methane releases from land fills can be harnessed for profit if the gas is collected and sold rather than simply allowed to disappear into the atmosphere. This is actually more practical than it might sound, for many land fill sites (see Figure 3[4]) are already equipped with tubes that allow methane to escape from deep within the land fill site. (If the methane is not allowed to escape, it can cause instabilities in the land fill and possibly explosions.)

[3] Environmental Protection Agency. *Addendum to U.S. Methane Emissions 1990-2020: Inventories, Projections and Opportunities for Reduction.* Washington, DC: National Center on Environmental Publications and Information, 2001.
[4] Image source: http://www.howstuffworks.com/landfill.htm