

### Homework Assignment 16: Due at the beginning of class 4/5/02

The mathematical content of this assignment consists of calculating average values of functions using areas under curves, integration and antiderivatives. The two contexts that you will apply this concept of average value to are blood pressure and ocean depth. In Questions 1, 2 and 3 you will calculate the average blood pressure of a rhesus monkey (*Macaca mulatta* – see Figure 5<sup>1</sup>) and in Questions 4 and 5 you will calculate the average depth of the Pacific Ocean. Perhaps surprisingly, both calculations will use exactly the same mathematical principles.

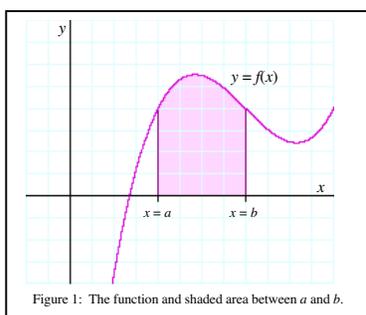


Figure 1 shows the graph of a function  $y = f(x)$ . The area beneath the curve between  $x = a$  and  $x = b$  has been shaded. Recall that the average value of the function  $f(x)$  between  $x = a$  and  $x = b$  is given by:

$$\text{Average value} = \frac{\int_a^b f(x) \cdot dx}{b - a}.$$

**If you are under a lot of time pressure at the moment, skip to page 4 of this assignment.**



Figure 2: Astronaut Donald K. “Deke” Slayton (1924-1993). (a) Slayton on Earth immediately before his flight on the Apollo-Soyuz Test Project in 1975. (b) Slayton while in orbit. Notice how much puffier his face appears, and in particular, how bulbous his nose has become while in space.

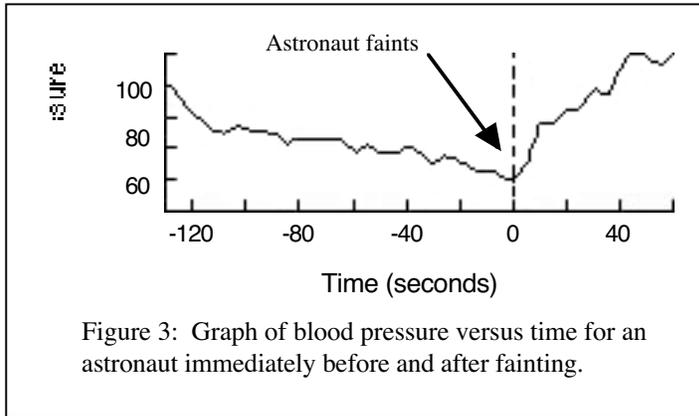
During the 1960’s, a few human beings spent extended periods of time in orbit around the Earth. While in orbit, the gravitational force experienced by these astronauts and cosmonauts was very small in comparison to the gravitational force that they experienced while on Earth. These space explorers also noticed changes in the way that their bodies functioned when they remained in orbit. Figure 2<sup>2</sup> shows astronaut Donald K. “Deke” Slayton (1924-1993) while on Earth and while in orbit

<sup>1</sup> Image source: <http://galileo.phys.virginia.edu/>

<sup>2</sup> Image sources: (a) <http://www.jsc.nasa.gov/er/she/slayton.htm>

(b) <http://lsda.jsc.nasa.gov/APOLLO/ASCREW.html>

around the Earth. While in orbit, Slayton's face has become noticeably swollen, and his nose has become very bulbous and pronounced. Astronauts also noticed that while in orbit their legs became much thinner than normal. This condition was unofficially called "puffy face and chicken leg syndrome."



When returning to Earth after protracted stays in orbit, astronauts also reported a near-insatiable thirst (even though they did not feel thirsty while still in orbit only one hour before). Perhaps most seriously, astronauts were also prone to fainting for a period of time after returning to the Earth.

Studies conducted by NASA (see Figure 3<sup>3</sup>) showed that immediately before fainting, the astronaut's blood pressure dropped suddenly. This was highly unusual as the astronauts were in peak physical condition and many had never fainted once in their entire lives. Clearly, exposure to extremely low gravity for a length of time produced some unexpected physiological effects that NASA scientists were at a loss to explain.

NASA scientists termed the cardiovascular condition that astronauts acquired while in a very low gravity (microgravity) environment *reduced orthostatic tolerance*. During the 1970's and 1980's unprecedented cooperation between the government of the former Soviet Union and NASA led to the "Cosmos" experiments. The goal of these experiments was to better understand the effects of microgravity on biological entities such as plants and animals. Between November 25, 1975, and January 10, 1993, a total of eight satellites were launched (see Figure 4<sup>4</sup>). The first satellite (Cosmos 782) carried an eclectic menagerie including rats, fruit flies, fish embryos and carrots<sup>5</sup>. Subsequent satellites carried mainly rats and rhesus monkeys (*Macaca mulatta* – see Figure 5).

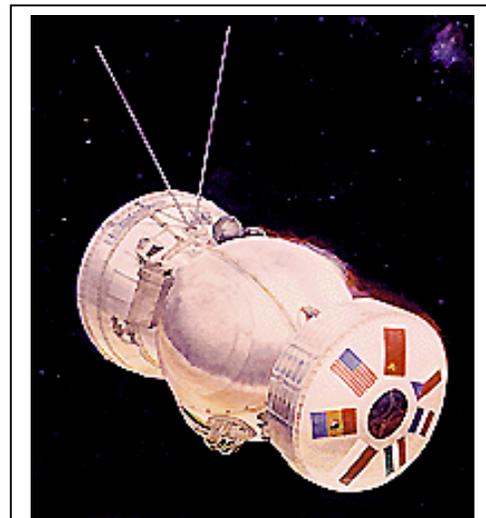


Figure 4: Artist's conception of one of the Cosmos satellites in orbit. These satellites were able to re-enter the Earth's atmosphere and return their animal passengers to Earth at the end of the experiment.

<sup>3</sup> Image source: [http://atrs.arc.nasa.gov/r\\_t/1996/img/02HEDS-Kuriyama.JPG](http://atrs.arc.nasa.gov/r_t/1996/img/02HEDS-Kuriyama.JPG) The data used in Figure 3 are taken from the report: K. Kuriyama, T. Ueno, R.E. Ballard, D.E. Watenpugh, S.M. Fortney and A.R. Hargens. (1996) "Cerebrovascular responses prior to fainting." NASA Technical Memorandum 112195.

<sup>4</sup> Image source: <http://lifesci.arc.nasa.gov/images/Cosmos.gif>

<sup>5</sup> Source: <http://neurolab.jsc.nasa.gov/ustime.htm>

The mission that you will study (Cosmos 1514, launched on December 14, 1983) carried two rhesus monkeys and four rat fetuses. The monkeys were outfitted with blood pressure monitors on various parts of their bodies to study how blood circulation in primates changed when they were exposed to a microgravity environment. NASA scientists hoped that the data collected from these monkeys would shed more light on the phenomenon of *reduced orthostatic tolerance* experienced by human space explorers when returning from orbit.



Figure 5: An adult rhesus monkey (*Macaca mulatta*).

Since the Cosmos 1514 experiment in 1983 a great deal of research has been done on changes to the function of the cardiovascular system<sup>6</sup> in humans and animals when exposed to a microgravity environment<sup>7</sup>. Much of this research was conducted aboard the Mir space station and during Space Shuttle missions. A consensus has emerged among researchers<sup>8</sup> that when exposed to a microgravity environment for a prolonged period of time, the human cardiovascular system changes in the following ways:

- **Elevated heart rate.** Astronauts' hearts beat faster when they are in orbit.
- **Increased blood pressure in the head and upper body.**
- **Reduced blood pressure in the extremities of the body, especially the feet.**
- **Reduced blood pressure in the veins.** In a microgravity environment the body does not have to work as hard to return the blood from the legs to the heart, as the blood does not have to be pumped up the legs against the force of normal Earth gravity.
- **Shifts in distribution of body fluids.** The main shift is that more fluids are located in the head and upper body, making the astronaut's faces appear "puffy" and their legs appear unusually thin.
- **Loss of blood plasma volume.** Astronauts may lose 20% or more of their blood.
- **Post-flight reduced orthostatic tolerance.**
- **Post-flight reduced exercise capacity.**

In Questions 1 and 2 of this homework assignment you will replicate some of the analysis that researchers did to establish that microgravity environments raise the blood pressure in the head and upper body. In Question 3 you will relate these calculations to the notion of average value that was introduced in class using integrals.

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<sup>6</sup> Other lines of research have concentrated on the effects of microgravity on muscles, the skeletal system, reproduction and the development of embryos.

<sup>7</sup> A strong motivation that has spurred much of this research is the (near future) prospect of long-term human space exploration, travel and possibly recreation. According to some reasonably credible experts, during your lifetime it may become possible for reasonably "ordinary" people to journey into space. Source: D. O'Neil, I. Bekey, J. Mankins, T.F. Rogers and E.W. Stallmer.. *General Public Space Travel and Tourism. Volume 1.* Washington, DC: Marshall Spaceflight Center, 1998.

<sup>8</sup> For example see the review article: V. Convertino and G.W. Hoffler. (1992) "Cardiovascular physiology. Effects of microgravity." *Journal of the Florida Medical Association*, **79**(8): 517-524.

In an adult rhesus monkey<sup>9</sup>, the blood pressure<sup>10</sup> in the arteries of the neck is usually between 60 mmHg and 80 mmHg with an average of about 72 mmHg. During the Cosmos 1514 experiment the rhesus monkeys each wore a blood pressure sensor around their neck. Figure 6 (below) shows a graph of blood pressure versus time for one of the monkeys.

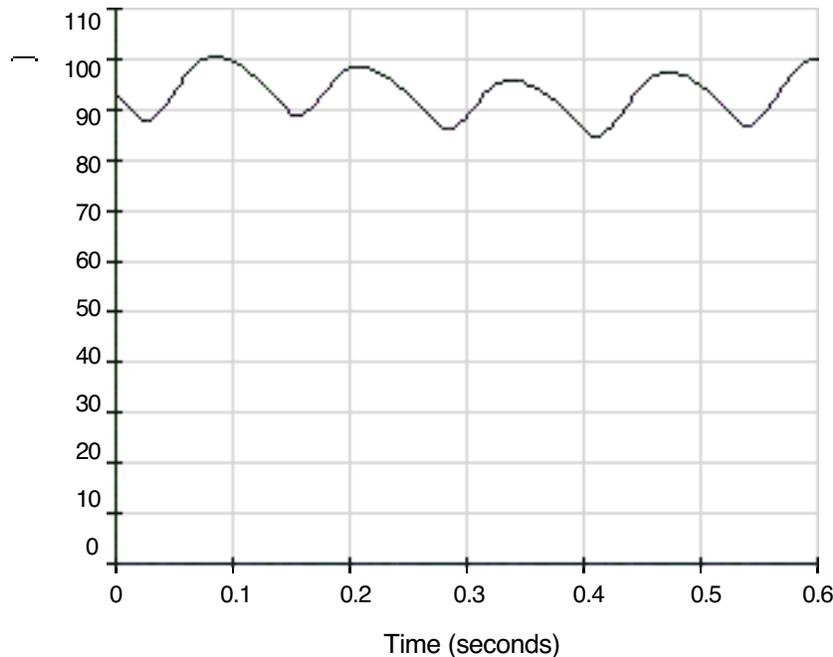


Figure 6: Graph showing blood pressure in neck arteries versus time for one of the rhesus monkeys on board the Cosmos 1514 satellite.

1. Estimate the area under the curve in Figure 6 between time  $T = 0$  and time  $T = 0.6$ .
2. Use your answer to Question 1 to estimate the average blood pressure in the arteries in the neck of the rhesus monkey over the time interval between  $T = 0$  and  $T = 0.6$ . Is your result consistent with the hypothesis that, when exposed to a microgravity environment, the blood pressure in the head and upper body of a primate rises?

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<sup>9</sup> The numbers, data and graphs that are used in Questions 1, 2 and 3 of this homework were obtained from the papers:

- H. Sandler. (1987) “Cardiovascular results from a rhesus monkey flown aboard the Cosmos 1514 spaceflight.” *Aviation, Space and Environmental Medicine*, **58**(6): 529-536.
- V.I. Korolkov et al. (1992) “Circulation in primates exposed to microgravity aboard the cosmos biosatellites.” *The Physiologist*, **35**(1): 245-247.

<sup>10</sup> The units of pressure used in this homework assignment are “mmHg.” This stands for “millimeters of mercury” which is a somewhat archaic unit of pressure that nevertheless appears to be very popular among space physiologists.

3. At the very beginning of this homework assignment, the average value of a function  $y = f(x)$  over the interval  $x = a$  to  $x = b$  was defined by the formula:

$$\text{Average value} = \frac{\int_a^b f(x) \cdot dx}{b - a}.$$

In a few sentences, explain how the calculations that you have done in Questions 1 and 2 correspond to this formula.

In Questions 4 and 5 of this homework assignment, you will use the concept of average value to calculate the average depth of the Pacific Ocean. Figure 7<sup>11</sup> shows a cross-section of the Pacific Ocean stretching from Japan on the left to California on the right. (The region of tall, narrow spikes near the center of Figure 7 is Hawaii.)

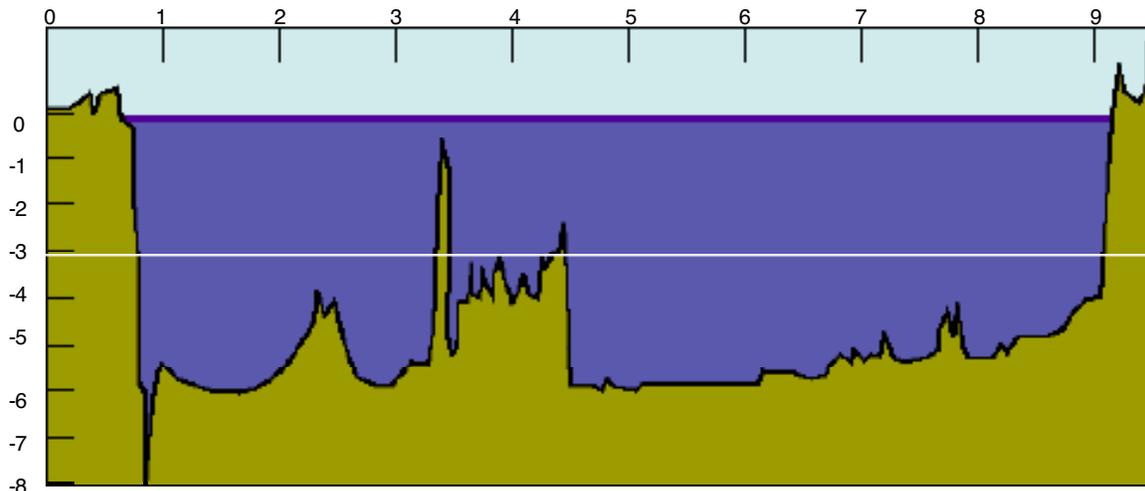


Figure 7: Cross-section of the Pacific Ocean. The numbers on the vertical axis are the depth in units of kilometers (km) and the numbers on the horizontal axis are the distance from Japan in units of thousands of kilometers.

4. Using the variables:

$x$  = Distance from Japan (in units of thousands of kilometers)  
 $y$  = Depth of Pacific Ocean (in units of kilometers)

the shape of the ocean floor shown in Figure 7 can be approximated by the equation:

$$y = p(x) = 0.048 \cdot x^4 - 0.9 \cdot x^3 + 5.79 \cdot x^2 - 14.89 \cdot x + 7.69.$$

<sup>11</sup> Image source: <http://geosun1.sjsu.edu/>

Find an equation for an antiderivative,  $P(x)$ , of  $p(x)$ . Your equation for  $P(x)$  should include one unspecified constant (i.e. remember the “+ $C$ ” !). If you were to evaluate the quantity

$$P(9) - P(0.7),$$

which of the following would you have calculated:

- The light blue area from Figure 7?
- The dark blue area from Figure 7?
- The tan area from Figure 7?

Indicate which of these areas is represented by the quantity  $P(9) - P(0.7)$  as part of your answer to this problem.

5. What is the average depth of the Pacific Ocean (in units of kilometers)? Show any relevant work that you used to determine your answer.