Homework Assignment 17: Due at the beginning of class 4/17/02

The mathematical content of this homework addresses two of the topics that you have studied. The first (represented by Questions 1, 2 and 3) is an example of a "slicing problem" in which you will set up and evaluate an integral to find the amount of sulfuric acid suspended in the atmosphere of the planet Venus. The second (Questions 4 and 5) concerns the equations that represent periodic functions.

If you are under a lot of pressure at the moment, skip to page 5 of this assignment.

Figure 1: Dr. Frank Drake.

As far as we know, the only place in the universe where life exists (or has ever existed) is the planet Earth. However, given the vastness of the cosmos many people (credible scientists among them) believe that there must be a high probability that life either exists, or has existed, elsewhere.

In 1961, astronomer Frank Drake (see Figure 1[1]) formulated an equation to calculate the number of "communicative civilizations" in the Milky Way galaxy – that is the number of civilizations that could produce radio signals detectable by radio telescopes on Earth.

The equation is usually stated:

$$N = R_* \cdot f_p \cdot n_e \cdot f_l \cdot f_i \cdot f_c \cdot L$$

where:
N = The number of communicative civilizations in the Milky Way galaxy.
R_* = The rate of formation of stars suitable for intelligent life.
f_p = The fraction of suitable stars with planetary systems.
n_e = The number of Earth-like plants per planetary system.
f_l = The fraction of Earth-like plants on which life develops.
f_i = The fraction of life-producing planets where intelligence develops.
f_c = The fraction of planets where technology develops.
L = The "lifetime" of a communicating civilization.

One of the points that Drake hoped to establish in formulating this equation is that there could be civilizations *broadcasting* radio signals . Hence our best chance of detecting

[1] Image source: http://www.pinnacelnews.com/

life elsewhere is to *listen* for extraterrestrial radio signals that are unusual and could possibly have been produced by an extraterrestrial civilization.

Drake's equation has been described[2] as one of the most important equations developed in the twentieth century, eclipsed only by Einstein's famous $E = mc^2$.

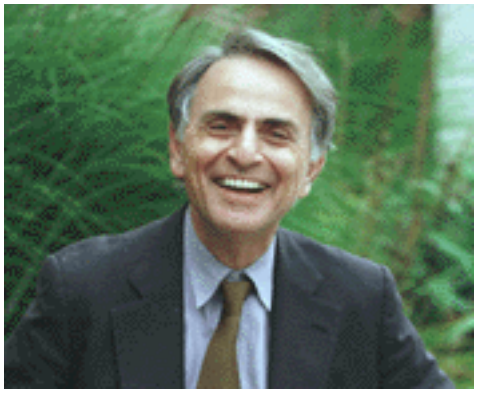

Figure 2: Dr. Carl Sagan (1934-1996).

Perhaps the most well-known[3] estimate of N was made by Carl Sagan (1934-1996, see Figure 2[4]). Sagan estimated that there could be as many as six million communicative civilizations in the Milky Way galaxy. More recent estimates[5] suggest a number closer to 1000.

At the time of writing, the only truly astonishing extraterrestrial radio signal that has been reported was received at about 11:16pm on August 15, 1977 in the unlikely location of Delaware, Ohio. Delaware, Ohio is the site of Ohio State University's "Big Ear" radio telescope. The signal (see Figure 3[6]) was noticed by Jerry Ehman., who wrote "Wow!" in the margin of the computer print-out. Ever since, this unique event has been known as the "WOW" signal.

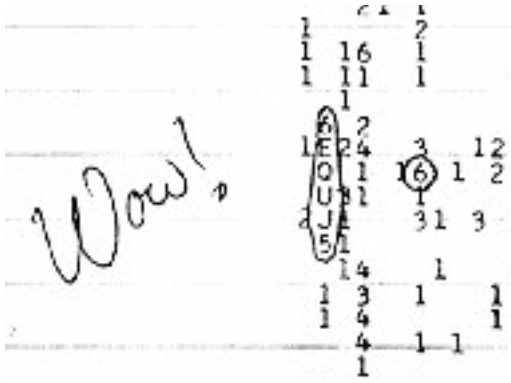

Figure 3: Part of the computer print-out from the Big Ear radio telescope, August 15 1977. The numbers represent the strength of readio signals received. When the telescope received the WOW signal, the strength of the signal was so high that the computer used letters (instead of numbers) to represent the signal.

New evidence for the possibility of life on other planets was unearthed in 1984 by NASA geologists excavating the Allan Hills in the western extremes of Antarctica (see Figure 4[7]).

Referred to as "ALH84001" the meteorite was the subject of a flurry of scientific and public interest when a team of scientists

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[2] Source: http://www.pbs.org/saf/1103/features/outthere.htm

[3] See: C. Sagan and F. Drake. (1975) "The search for extraterrestrial life." *Scientific American*, May 1975: 809-89.

[4] Image source: http://www.carlsaganfoundation.org/

[5] See: I. Stewart. (2000) "Where are they? Maybe we are alone in the galaxy after all." *Scientific American* , July 2000: 38-43.

[6] Image source: http://www.bigear.org/

[7] Image source: http://www.jpl.nasa.gov/

published an article[8] in the journal *Science* describing microscopic formations and chemical residues within ALH84001. The scientists suggested that these microscopic structures could be fossilized bacteria, or the fossilized remains of organic matter created by bacterial action. The article ended with the sentence:

> "Although there are alternative explanations for each of these phenomena taken individually, when they are considered collectively, particularly in view of their spatial association, we conclude that they are evidence for primitive life on Mars." (p. 930)

In essence, what the researchers were saying was that although each of the features (see Figure 5[9]

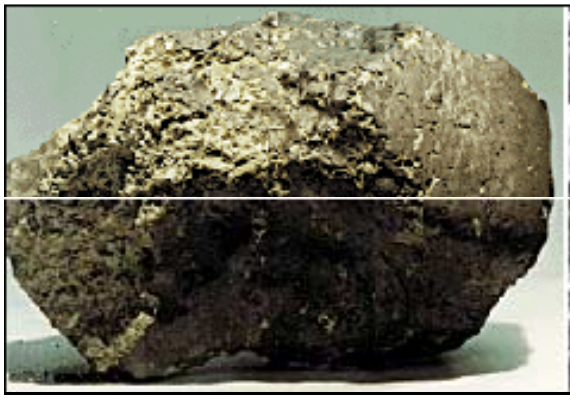

Figure 4: The Martian meteorite ALH84001.

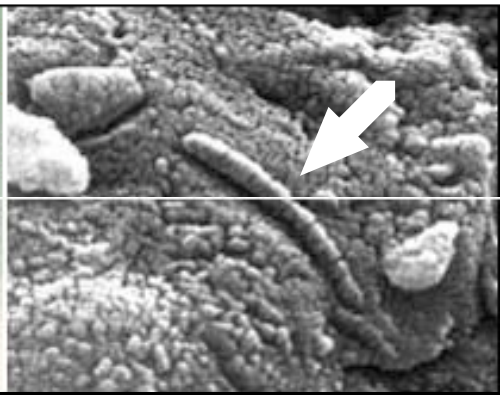

Figure 5: Electron micrograph of part of the Martian meteorite, ALH84001. Note the long tube-like formation that was cited as possible evidence for primitive life.

Equations, radio telescopes, electron micrographs, hexadecimal notation and computer printouts have the potential to provide compelling evidence for people interested in the scientific study of the possibility of extraterrestrial life. However, in the opinion of others, there is little need for the slow accumulation of scattered fragments of such circumstantial evidence. In the view of some, extraterrestrial life not only exists, but extraterrestrials have been visiting Earth for some time.

One fairly well-known proponent of these views is Frank Stranges[10] (see Figure 6 [11]), president of the "National Investigations Committee on UFOs," as well as the religious

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[8] D.S. McKay, E.K. Gibson, K.L. Thomas-Keprta, H. Vali, C.S. Romanek, S.J. Clemett, X.D.F. Chiiler, C.R. Maechling and R.N. Zare. (1996) "Search for past life on Mars: Possible relic biogenic activity in Martian meteorite ALH84001." *Science*, **273**(5277): 924-930.

[9] Image source: http://fyi.cnn.com/

[10] On the back cover of his book *The UFO Conspiracy*, Stranges claims to hold a doctorate in International Law from Union University suggesting that he might be properly addressed as Dr. Frank Stranges. However, routine inquiries (see http://www.ufomid.com/misc/1997/feb/d02-001.shtml) to all of the Union Universities that Stranges might have attended revealed that either the university in question does not have a law school, or else that the law school does not offer any degrees in "International Law."

[11] Images sources: (a) http://www.flyingsaucers.com/stranges.htm

organizations, "International Evangelical Crusades" and "International Theological Seminary of California." Frank Stranges claims[12] that in 1956 he had repeated encounters with UFOs and, in 1957, was introduced to a being from another planet, This being was named "Valiant Thor" and according to Stranges, Thor was a Venusian "Grand Space Commander" who "…had no fingerprints and could read minds." According to Stranges, on March 16, 1957, Grand Space Commander Valiant Thor landed his flying saucer on the outskirts of Alexandria, VA.

Figure 6: Frank E. Stranges

Figure 7: Alleged Venusian Grand Space Commander, Valiant Thor.

Understandably nonplussed by this interplanetary spectacle, two officers of the Alexandria police force advanced on Grand Space Commander Valiant Thor with their pistols drawn. Using his powers of thought transference, Grand Space Commander Valiant Thor was able to not only escape a hail of law-enforcement hot lead, but was also able to convince the police officers to take him to Washington, DC, in their patrol car. When he arrived in DC, Grand Space Commander Valiant Thor was met by the Secretary of Defense. Evidently realizing that he was out of his depth, the Secretary quickly ushered Grand Space Commander Valiant Thor to the White House to meet with President Dwight D. Eisenhower and Vice President Richard M. Nixon. Evidently, once inside the Oval Office, Thor outlined a plan by which everyday people could improve their lives in simple, practical ways[13].

Although fantastic reports of encounters with unearthly beings are particularly remarkable, what is unusual about this case is that "Valiant Thor" was *photographed* on a regular basis (see Figure 7[14]) and has *published* at least one book, the 1993 treatise *Outwitting Tomorrow*.

In this homework assignment, you will show that the planet Venus is a very unlikely home for a race of technologically advanced (or any) beings. When you complete Questions 1, 2 and 3 you will be able to see that Venus has so much sulfuric acid in its

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[12] See, for example: F.E. Stranges. *Stranger at the Pentagon. A True Life Story*. (5th Revised Edition), Van Nuys, CA:Universe Publishing Company, 1997. You can obtain a copy of the book by writing to Stranges at: P.O. Box 73, Van Nuys, CA 91408.

[13] See: V. Thor. *Outwitting Tomorrow*. Van Nuys, CA: Universe Publishing Company, 1993.

[14] Image source: http://www.algonet.se/

atmosphere that it seems highly unlikely that any creatures of flesh could have evolved or survive on the planet at all[15].

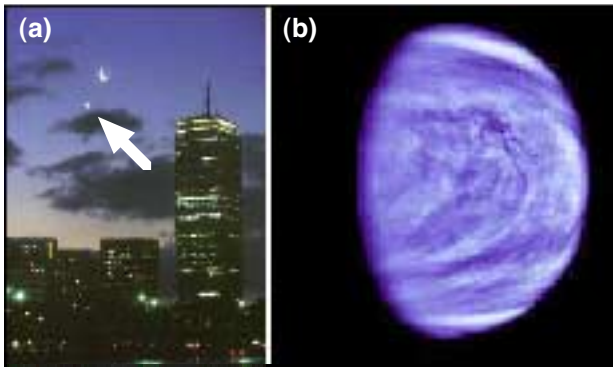

Figure 8: The planet Venus. (a) Even without magnification, Venus is often visible as here above the Boston skyline. (b) Venus from space.

The planet Venus (see Figure 8[16]) is surrounded by a dense atmosphere comprised mainly of carbon dioxide (96%) with traces of nitrogen (3%) and water vapor (0.003%)[17]. As is shown in Figure 8(b), the atmosphere of Venus has clouds that appear (from a distance) to resemble clouds on the Earth. The clouds of Venus are made up of droplets of concentrated sulfuric acid (H_2SO_4). The cloud layer begins about 30km above the surface of Venus, and ends at about 60km above the surface. Below the cloud layer (0-30km above the surface of Venus) the atmosphere includes a "haze" of sulfuric acid droplets.

1. Figure 9[18] shows a cross-sectional view of part of the Venusian atmosphere. The piece of the Venusian atmosphere represented by the large shaded "ring" in Figure 9 is at a height of x km above the surface of the planet, and has a thickness of dx kilometers. In a sentence or two describe the appearance of this "slice" of the Venusian atmosphere and find a formula for its volume. The radius of the planet Venus is approximately 6052 km.)

2. Figure 10[19] shows the concentration of sulfuric acid in the Venusian atmosphere as a function of altitude for the cloud layer (30-60 km above the surface). The concentration of sulfuric acid in the "haze layer" (0-30 km above the surface) is about 35.68 metric tons per cubic kilometer. (The units of the concentration are

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[15] These arguments and calculations don't disprove the possibility of life on Venus, but it is hard to imagine what kind of life form could have evolved under the very harsh Venusian conditions. Some "believers" (who have limited credibility, even by the standards of the UFO subculture) contend that there are beings who live on Venus, but they are "spirit beings" who lack physical form and are therefore not bothered by the harsh Venusian environment. Perhaps the most famous of these individuals is "Omnec Omec" who claims to be a 246 year old Venusian who came to Earth in 1955. Although one might imagine a spirit-being from Venus to live an unusual life, Omnec Onec appears to have been content to move to Chicago, marry, raise three children and supplement her family's income with occasional work as a bar-maid, clothes designer and cashier.

[16] Image source: (a) http://www.mos.org/ (b) http://www.nasa.gov/

[17] Source: http://nssdc.gsfc.nasa.gov/planetary/factsheet/venusfact.html

[18] This diagram was made with an image from: http://www.nasa.gov/

[19] The data used to construct Figure 10 is taken from: M.A. Kolodner and P.G. Steffes. (1998) "The microwave absorption and abundance of sulfuric acid vapor in the Venus atmosphere based on new laboratory measurements." *Icarus*, **132**(3): 151-169.

always metric tons per cubic kilometer in this homework assignment.) Set up two integrals, one that gives the amount of sulfuric acid (in units of metric tons) in the haze layer and one that gives the amount of sulfuric acid (in units of metric tons) in the cloud layer of Venus.

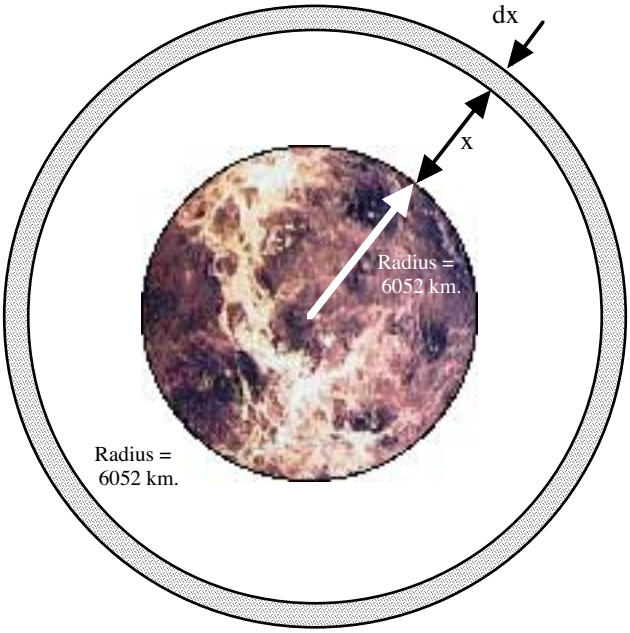

Figure 9: Cross-sectional view of a "slice" of the Venusian atmosphere. The slice shown is at a height of "x" km above the surface of the planet, and has a thickness of "dx."

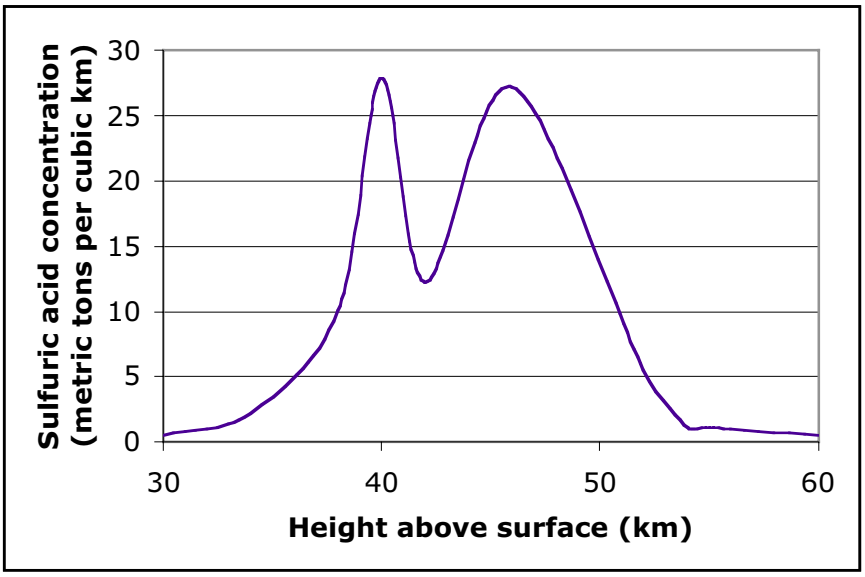

Figure 10: Concentration of sulfuric acid in Venus cloud layer.

The curve in Figure 10 is approximated by the equation:

$$y = 0.001 \cdot x^4 - 0.26 \cdot x^3 + 16.74 \cdot x^2 - 472 \cdot x + 4886.48.$$

3. Calculate the numerical value of the integral that you have set up for the amount of sulfuric acid in the haze layer. Describe (and perform) a simple calculation - based on geometry rather than calculus – that will enable you to check your answer to this problem. In a sentence or two, explain why you could not use a similar, simple calculation to check your answer if you had been asked to work out how much sulfuric acid was in the cloud layer of Venus.

In Questions 4 and 5 you will set up equations for two periodic functions. Some of the data that you will need to do this is given in Table 1 (below)[20]. In Table 30, January 1 is "Day 0" of the year.

Day of the Year	Hours of Daylight	Day of the Year	Hours of Daylight
0	9.1	210	14.6
30	9.9	240	13.3
60	11.2	270	11.9
90	12.7	300	10.6
120	14	330	9.5
150	15	360	9.1
180	15.3		

Table 1: Daylight hours during 1998 for Cambridge, MA.

4. Use the data in Table 1 to plot a graph showing Hours of Daylight versus Day for Cambridge, MA. Calculate the period, midline and amplitude, and show how these quantities appear on your graph. Find an equation for the number of hours of daylight as a function of the day for Cambridge, MA.

5. Figure 11[21] shows the location of the city of Cambridge, New Zealand. Use the data given in Table 1 to sketch a plausible graph showing Hours of Daylight versus Day for Cambridge, New Zealand. Find an equation for the number of hours of daylight as a function of the day for Cambridge, New Zealand.

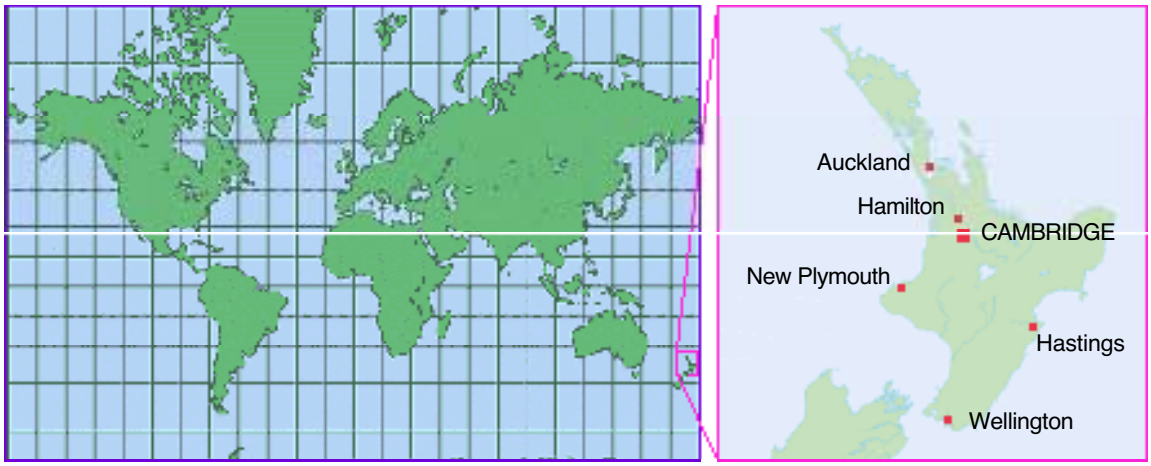

Figure 11: Location of Cambridge, New Zealand. Note that the country of New Zealand is located in the Southern Hemisphere.

[20] Source of data: The Texas Education Network, Texas Essential Knowledge and Skills, Mathematics Module: Oscillations.

[21] Image sources: http://www.kart-a-bag.com and http://www.maritimeheritage.org/