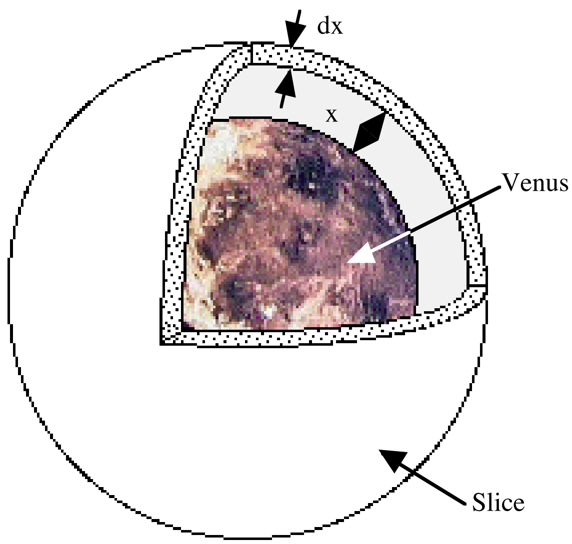
Homework Assignment 17: Solutions

1. The "slice" of the Venusian atmosphere shown in Figure 9 on the homework assignment only reveals a two-dimensional cross-section of the slice and not the three-dimensional shape of the slice itself.

Imagine a basketball. The basketball consists of a thin rubber skin with air inside it. The structure of the slice of Venusian atmosphere is like the thin rubber skin of the basketball. (See diagram below.)



The volume of the three-dimensional slice will be given by[1] the surface area of the inside of the spherical shell multiplied by the thickness of the shell. The thickness of the shell is given by *dx* (see Figure 9 on the homework assignment) and the radius of the shell (from the center of Venus) is given by:

Radius = (Radius of Venus) + (Height above surface of Venus) = 6052 + *x*.

Therefore, the volume of the slice will be:

$$\begin{aligned} \textit{Volume of slice} &= 4\pi \cdot (\textit{Radius})^2 \cdot dx \\ &= 4\pi \cdot (6052 + x)^2 \cdot dx \end{aligned}$$

[1] Note that the formula given here is not, strictly speaking, the actual volume of the slice. The formula that is given here is a *very* close approximation to the volume of the slice, which works well when the thickness of the slice, *dx*, is a very small, positive number.

2. The limits of integration for each of the integrals will be the heights above the surface of Venus where the "cloud" and "haze" layers of the atmosphere begin and end. The limits of integration are summarized in Table 1 (below).

Layer	Lower limit of integration	Upper limit of integration
Haze	0	30
Cloud	30	60

Table 1: Upper and lower limits of integration for integrals that will represent the amount of sulfuring acid (in units of metric tons) in the layers of the Venusian atmosphere.

The quantity to be integrated is the amount (in units of metric tons) of sulfuric acid (H_2SO_4) contained in the slices of Venusian atmosphere. Using the usual reasoning:

$$Amount = (Concentration)\times(Volume).$$

Therefore, for the **haze layer** where the concentration of sulfuric acis is always equal to 35.68 metric tons per square kilometer, the quantity to be integrated will be:

$$Amount = 4\pi \cdot (35.68) \cdot (6052 + x)^2 \cdot dx\,.$$

In the **cloud layer**, the concentration of sulfuric acid was given by the equation:

$$Concentration = 0.001 \cdot x^4 - 0.26 \cdot x^3 + 16.74 \cdot x^2 - 472 \cdot x + 4886.48.$$

Therefore, for the **cloud layer**, the quantity to be integrated is:

$$Amount = 4\pi \cdot \left(0.001 \cdot x^4 - 0.26 \cdot x^3 + 16.74 \cdot x^2 - 472 \cdot x + 4886.48\right) \cdot (6052 + x)^2 \cdot dx$$

Putting the limits of integration together with the quantities to be integrated gives the following integrals for amount of sulfuric acid (in units of metric tons):

Haze Layer:

$$\int_0^{30} 4\pi \cdot (35.68) \cdot (6052 + x)^2 \cdot dx$$

Cloud Layer:

$$\int_{30}^{60} 4\pi \cdot \left(0.001 \cdot x^4 - 0.26 \cdot x^3 + 16.74 \cdot x^2 - 472 \cdot x + 4886.48\right) \cdot (6052 + x)^2 \cdot dx$$

3. Before trying to find an antiderivative, it is a wise precaution to try to simplify the expression that you are trying to integrate as much as is algebraically possible.

$$\int_0^{30} 4\pi \cdot (35.68) \cdot (6052 + x)^2 \cdot dx = 4\pi \cdot (35.68) \cdot \int_0^{30} \left(36626704 + 12104 \cdot x + x^2\right) \cdot dx.$$

Calculating the antiderivative gives that the amount of sulfuric acid in the haze layer will be given by:

$$\begin{aligned} \textit{Amount} &= 4\pi \cdot (35.68) \cdot \left[36626704 \cdot x + 6052 \cdot x^2 + \tfrac{1}{3} \cdot x^3\right]_0^{30} \\ &= 4.95 \times 10^{11} \text{ metric tons.} \end{aligned}$$

As the concentration is constant (or close to constant) throughout the haze layer, another way to calculate the mass of sulfuric acid in the haze layer is to calculate the volume of the haze layer and multiply this by the (constant) concentration of 35.68 metric tons per cubic kilometer.

The volume of the haze layer is the difference between the volumes of spheres with radii of 6082km and 6052km. Calculating this difference in volumes using the volume formula for a sphere of radius r:

$$\textit{Volume of sphere} = \frac{4}{3}\pi \cdot r^3,$$

Gives:

$$\textit{Volume of Haze Layer of Venus} = \frac{4}{3}\pi \cdot \left[6082^3 - 6052^3\right] \approx 1.38765 \times 10^{10} \text{ (km)}^3.$$

Multiplying this volume by the concentration of sulfuric acid in the haze layer gives:

$$\textit{Amount of sulfuric acid} = (35.68) \cdot (1.38765 \times 10^{10}) \approx 4.95 \times 10^{11} \text{ metric tons.}$$

The one fact that made this calculation work was that the concentration of sulfuric acid was constant throughout the haze layer of the Venusian atmosphere. As Figure 10 from the homework assignment showed, the density of sulfuric acid was definitely not constant in the cloud layer of the Venusian atmosphere. Therefore, a similar calculation (which uses the constancy of the concentration to enable you to calculate the volume separately and then simply multiply by the concentration) will not be appropriate.

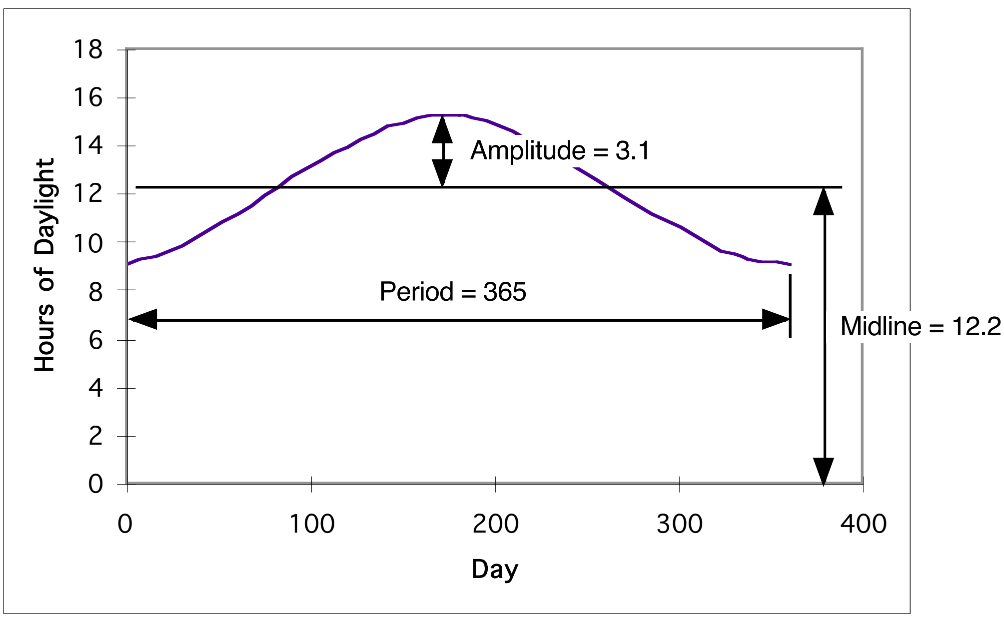
4. The graph of the daylight hours versus days for Cambridge, MA, is shown below. Also displayed on this graph are the period (365 days), the midline and the amplitude.

The **midline** may be calculated by averaging the highest and the lowest values of the function. Using the data give in Table 1 of the homework assignment, the midline will be:

$$Midline = \frac{15.3 + 9.1}{2} = 12.2.$$

The **amplitude** may be calculated by halving the difference between the highest and lowest values of the function, Using the data given in Table 1 of the homework assignment, the amplitude will be:

$$Amplitude = \frac{15.3 - 9.1}{2} = 3.1.$$



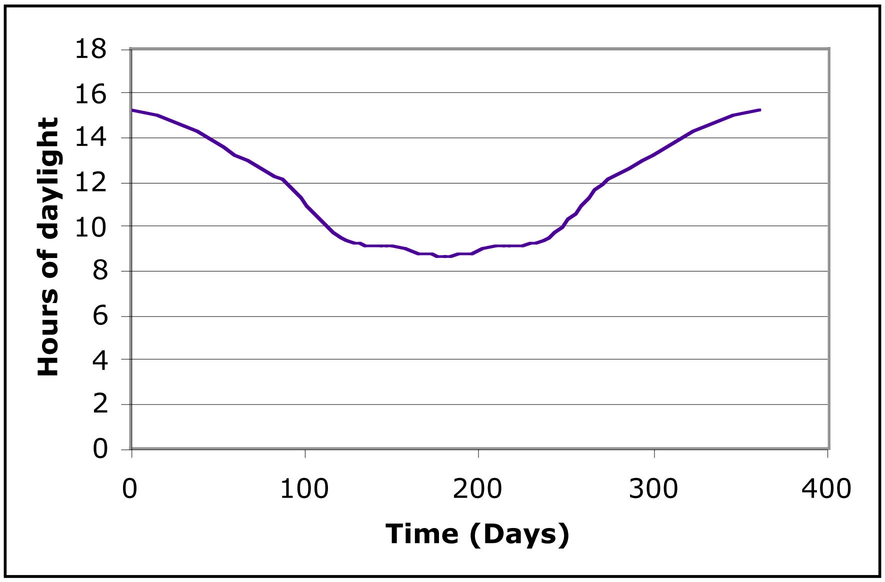
Based on the appearance of the graph shown above, the form of equation that will probably do the best job of representing the length of the day for Cambridge, MA, is:

$$f(x) = -A \cdot \cos\left(\frac{2\pi}{P} \cdot x\right) + M.$$

Substituting the values for the period, midline and amplitude determined above into this equation gives:

$$f(x) = -3.1 \cdot \cos\left(\frac{2\pi}{365} \cdot x\right) + 12.2.$$

5. As Cambridge, New Zealand, is located in the Southern hemisphere, the longest days of the year are in December and January, while the shortest days of the years are in June and July. A plot of length of day versus year will then look something like the graph shown below.



The equation for this graph (using the same period, midline and amplitude as the function for Cambridge, MA) will be:

$$g(x) = 3.1 \cdot \cos\left(\frac{2\pi}{365} \cdot x\right) + 12.2.$$