Homework Assignment 18: Due at the beginning of class 4/19/02.

The mathematical content of this homework assignment is in two parts. In the first part (Questions 1, 2 and 3) you will set up a "slicing problem" to calculate the total amount (biomass) of fish in a lake. In the second part of the homework assignment, you will use some of the regression techniques that you learned in Math Xa to explore some of the consequences that your result for total biomass suggests.

The introduction to these problems (which describes some of the geological and social history of the lake, as well as describing some of the methods used to collect the data that this assignment is based upon) can be downloaded separately.

NOTE: In several problems that you have completed over the last few weeks, depth was represented by a negative number. In this homework assignment, you should regard a depth of (say) 3km as $x = +3$. That is, depth should be regarded as a positive number in this homework assignment.

1. The density of fish biomass[1] in the Scottish lake Loch Ness is approximated by the function $p(x)$ defined by the equation given below. The units of this function are metric tons per cubic kilometer, and the variable x represents the depth (in units of kilometers).

$$p(x) = -188323018.5 \cdot x^4 + 14714825.03 \cdot x^3 - 368440.0735 \cdot x^2 + 3174.116 \cdot x + 0.1$$

Figure 1 (below) shows a three dimensional view of Loch Ness[2]. As shown in Figure 1, the three dimensional volume of Loch Ness can be sliced into horizontal slabs. The thickness of each slab is dx, and the area of each slab, $a(x)$, is given by the equation:

$$a(x) = 58.88 \cdot (1 - x) \text{ square kilometers.}$$

---

[1] This function was obtained from data included in: (I) J. Kubecka, A. Duncan and A.J. Butterworth. (1993) "Organisms detected in Loch Ness by dual-beam acoustics." *Scottish Naturalist*, **105**: 175-193. (II) A.J. Shine, D.S. Martin and R.S. Marjoram. (1993) "Spatial distribution and diurnal migration of the pelagic fish and zooplankton in Loch Ness." *Scottish Naturalist*, **105**: 194-238.

[2] This model for the physical shape of Loch Ness is an approximation based on the classic work: J. Murray and L. Pular (eds.) *Bathymetrical Survey of the Scottish Fresh-Water Lochs. Volume 1.* Edinburgh, Scotland: Challenger Office Publications.

Given the algebraic structure of the function $p(x)$, explain it is reasonable to slice Loch Ness up into horizontal slabs in order to calculate the total fish biomass in the Loch.

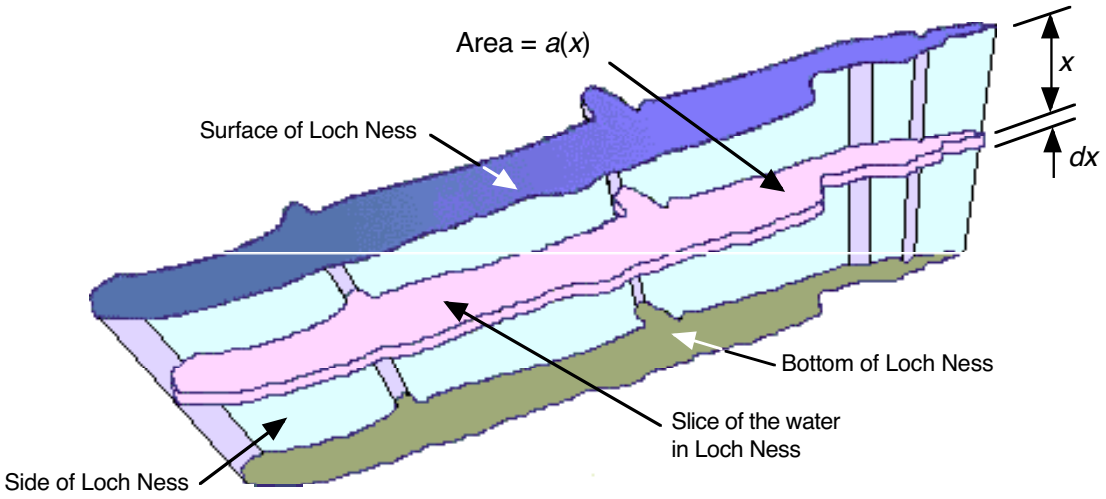

Figure 1: Three-dimensional model of Loch Ness. The pink slice in the middle of the diagram is at a depth of x, has an area of $a(x)$, and a thichness of dx.

2. Find a formula for the volume of one of the horizontal slabs shown in Figure 1. According to scientific explorations of the Loch[3], almost all of the fish in the Loch are found between the surface ($x = 0$) and a depth of 40 meters ($x = 0.04$). Set up an integral that will give the total fish biomass (in units of metric tons) for Loch Ness.

3. The following algebraic fact has been provided for your computational convenience – **there is absolutely no need for you to work out the following fact yourself.**

$$p(x)\cdot a(x) = (1.109\times10^{10})\cdot x^5 - (1.195\times10^{10})\cdot x^4 + (8.881\times10^{8})\cdot x^3 - (2.188\times10^{7})\cdot x^2 + (1.869\times10^{5})\cdot x + 5.888$$

Use the algebraic fact provided above to find the numerical value of the integral that you set up in Question 2. The number that you get will be the total fish biomass in Loch Ness (expressed in units of metric tons).

[3] For example, see: C.W. Beam, I.J. Winfield and J.M. Fletcher. "Stock assessment of the arctic charr (*Salvelinus alpinus*) population in Loch Ness." In I.G. Coux (ed.) *Stock Assessment in Inland Fisheries*. Oxford, England: Fishing News Books, 1996.

4. Table 1 (see below) gives the length, mass and typical daily food requirements for a number of aquatic animals. In this question you are going to create two equations:

- The first equation will give daily food consumption as a function of body mass.
- The second equation will give daily food consumption as a function of body length.

To create each equation:

i. Enter the relevant data from Table 1 into your calculator.
ii. Use the STATPLOT feature of your calculator to display the data graphically.
iii. Based on the appearance of the STATPLOT, decide what kind of function would do a reasonable job of representing the relationship shown in your STATPLOT.
iv. Use the regression features of your calculator to find an equation to represent the relationship.

Common name	Scientific Name	Typical length (meters)	Typical mass (kg)	Typical daily food comsumption (kg)
Bottlenose dolphin[4]	*Tursiops truncatus*	2.8	230	10.3
Harbor porpoise[5]	*Phocoena phocoena*	1.5	55	4.5
New Zealand Fur Seal[6]	*Arctocephalus forsteri*	1.6	160	11.2
Nurse shark[7]	*Ginglymartoma taurus*	2.4	150	13.5
Polar bear[8]	*Ursus maritimus*	2.0	680	11.4

Table 1: Body size, mass and daily food requirements for aquatic animals.

5. Table 2 (see next page) provides some information based on rumor, speculation an archaeology. All of the creatures (see Figure 2[9] for illustrations) listed in Table 2 have been suggested as possible candidates for Loch Ness monsters. Use the equation that you found in Question 4 to predict the daily food requirements for all of these creatures.

---

[4] Source: National Oceanic and Atmospheric Administration. "Backgrounder on Bottlenose Dolphins in the Shrewsbury River." National Marine Fisheries Service News, November 8 2000.
[5] Source: R. A. Kastelein, J. Hardeman and H. Boer. "Food consumption and body weight of harbor porpoises (Phocoena phocoena)." in A. J. Read, P. R. Wiepkema and P. E. Nachtigall (eds.) "The Biology of the Harbor Porpoise." De Spil D. V. Publisher: Woerden, The Netherlands, 1997.
[6] Sources: (1) R. H. Taylor. "New Zealand fur seals at the Bounty Islands." *New Zealand Journal of Freshwater and Marine Research*, **16**: 1-9, 1982. (2) P. W. Carey. "Fish prey species of the New Zealand fur seal." *New Zealand Journal of Ecology*, **16**: 41-46, 1992. (3) G. A. Knox. "The Biology of the Southern Ocean." Cambridge University Press: Cambridge, UK, 1994.
[7] Sources: (1) J. I. Castro. "The Sharks of North American Waters." College Station, TX: Texas A&M University Press, 1983. (2) The University of Michigan Museum of Zoology. (3) http://www.bio.metu.edu.tr/~serdar/food.html
[8] Source: National Council of Teachers of Mathematics. http://www.nctm.org/
[9] Image sources: (a) http://www.battleduck.com/pages/imagepages/dinoimagepages/elasmo.html (b) http://www.geocities.com/Area51/Station/8390/carc.html (c) http://www.pbs.org

Common name	Scientific name	Typical length (meters)	Typical mass (kg)
Elasmosaurus[10]	*Woolungo-saurus glendowerensis*	13.7 (body and neck) 5.7 (body only)	2320
Rotting carcass recovered by Zuiyo-Maru[11]	Unknown	9.6	1800
Nessie	*Nessiteras rhambopteryx*[12]	4.5[13]	Unknown

Table 2: Body dimensions for creatures suggested as candidate Loch Ness monsters.

Figure 2: Creatures that have been suggested as candidate Loch Ness Monsters. (a) The marine reptile *elasmosaurus*. (b) A rotting carcass caught in the nets of the Japanese fishing vessel Zuiyo Maru. (c) Respected British naturalist Sir Peter Scott's artisitc impression.

According to Sheldon and Kerr[14] a group of large, predatory aquatic animals living in a lake the size of Loch Ness would need at least 10-20 individuals in order to maintain a viable breeding population. Clearly this number is somewhat speculative, but it reinforces the point that in order for creatures to survive for hundreds of years in the Loch, there probably needs to be a breeding population with several individuals. Compare the numbers that you have just calculated to the fish biomass that you calculated in Question 3. From the point of view of food availability, could Loch Ness possibly support a population of large animals? In a sentence or two, briefly explain your reasoning.

---

[10] Source: Carnegie Library of Pittsburgh, http://www.clpgh.org/

[11] While operating off the coast of Christchurch, New Zealand on April 10 1977, the Japanese fishing vessel *Zuiyo-Maru* dredged up a large, decomposing carcass in its nets. The carcass was badly decayed and had such an offensive odor that the captain of the ship ordered it dumped overboard immediately. All that remains are a few photographs of the carcass taken by a crewman. On the basis of these photographs, many scientists have suggested the theory that the carcass was the remains of a dead basking shark, *Cetorhinus maximus*.

[12] This is the name proposed by Dr. Robert Rines and Sir Peter Scott for the animal, should it actually be proven to exist.

[13] The length given here is the approximate length of a large underwater object that was detected by sonar during an expedition in 1997 led by Dr. Robert Rines and Charles Wyckoff. Eye-witness accounts of the length of the Loch Ness monster vary considerably.

[14] Sheldon, R. W. and S. R. Kerr. (1972) "Density of monsters in Loch Ness." *Limnology and Oceanography*, **17**: 746-798.