Homework Assignment 19: Solutions

1. The completed version of Table 1 is given below. It is not that important that your answers agree precisely with the figures given here, although your answers should be close to these values.

Quantity	Measured value (give units)
Apparent length of left foot (Fig. 2)	12 mm
Apparent length of shin (knee to ground) (Fig. 2)	13 mm
Apparent length of thigh (waist to knee) (Fig 2.)	12.5 mm
Apparent height of upper body (waist to top of head) (Fig. 2)	24 mm
Angle between thigh and vertical (Fig. 3)	$\frac{\pi}{4}$ radians (or 45 degrees)
Angle between upper body and vertical (Fig. 3)	0.43633 radians (or 25 degrees)

Table 1: Measure these quantities from Figures 2 and 3 and record your results here.

2. The length of the footprints found by Roger Patterson and Bob Gimlin at Bluff Creek in 1967 measured 14.5 inches long. Therefore, the 12 mm image of the sole of the left foot shown in Figure 2 of the homework assignment is an image of a foot that is 14.5 inches long. We can use this ratio to calculate the actual lengths of the other quantities listed in Table 2.

For example, to calculate the actual length of the upper body (which was measured to be 24 mm in Figure 2), we can simply make a ratio:

$$\frac{\text{Actual length of upper body}}{\text{Length of upper body from Fig. 2}} = \frac{\text{Actual length of foot}}{\text{Length of foot from Fig. 2}}.$$

Substituting the numbers from Table 1 and the actual length of the foot (14.5 inches) into this ratio and solving for the "Actual length of the upper body" gives:

$$\textit{Actual length of upper body} = \frac{24}{12} \cdot 14.5 = 29 \text{ inches.}$$

Similar ratio calculations for the thigh and shin give the results shown in the completed version of Table 2 (below).

Quantity	Value (inches)
Actual length of left foot	14.5
Actual length of shin (knee to ground)	15.708
Actual length of thigh (waist to knee)	15.1041
Actual length of upper body (waist to top of head)	29

Table 2: Actual dimensions for subject of Patterson film.

The height of the subject in the Patterson film will be $(15.708 + 15.1041 + 29) \approx 59.81$ inches. This is slightly less than five feet tall.

3. First, note that the shin of the subject (see Figure 3 in the homework assignment) in the Patterson film is close to vertical when the subject is walking. Therefore, the length that we have determined in Question 2 (15.7 inches) is the length that the shin will contribute to the height of the subject if the subject were to stand upright.

What remains is to use trigonometry to determine how much length the upper body and thigh would contribute to the height of the subject, were the subject to stand erect.

The Upper Body

Let U represent the "diagonal length" of the upper body of the subject. Figure 1[1] shows how a right angle triangle can be created using Frame 352 from the Patterson film, the angle measurements recorded in Table 1 and the lengths recorded in Table 2.

Using the definition of the sine function from triangle trigonometry in the right angle triangle from Figure 1 gives that:

$$\frac{\text{Opposite}}{\text{Hypoteneuse}} = \frac{29}{U} = \sin\left(65^\circ\right).$$

Solving for U and evaluating $\sin(65^\circ)$ on a calculator gives that the "diagonal length" of the upper body for the subject in the Patterson film is:

$$U = 31.9979 \approx 32 \text{ inches.}$$

[1] This diagram was created using an image from the Bigfoot Field Research Organization, http://www.bfro.net/

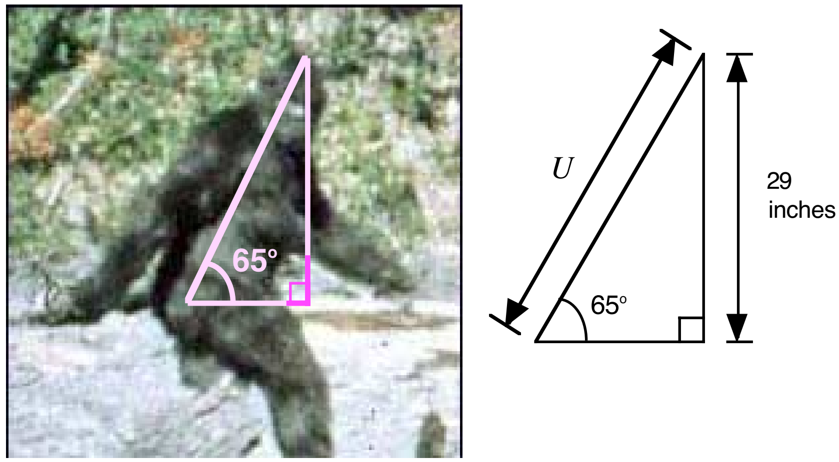

Figure 1: Creating a right angle triangle that can help you to calculate the diagonal length of the upper body for the subject of the Patterson film.

The Thigh

Let T represent the "diagonal length" of the thigh (in units of inches). Figure 2[2] shows how a right angle triangle can be created using Frame 352 from the Patterson film, the angle measurements recorded in Table 1 and the lengths recorded in Table 2.

Using the definition of the sine function from triangle trigonometry in the right angle triangle shown in Figure 2 gives that:

$$\frac{\text{Opposite}}{\text{Hypoteneuse}} = \frac{15.1}{T} = \sin\left(45^{\circ}\right).$$

Rearranging this equation to make T the subject and evaluating $\sin(45^{\circ})$ on a calculator gives that:

$$T \approx 21.3604 \text{ inches.}$$

If the subject in the Patterson film was to stand up straight, the subject would have a height of about (15.7 + 21.4 + 32) = 69.1 inches, or about five feet nine inches. Figure 3[3] shows a familiar creature with a height (when standing straight) of approximately five feet nine inches so that you can get a clear sense of just how

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[2] This diagram was created using an image from the Bigfoot Field Research Organization, http://www.bfro.net/

[3] Photo Credit: Professor William Stein, Department of Mathematics, Harvard University, One Oxford Street, Cambridge, MA 02138. Elements of this image were adapted from Frame 352 of the Patterson film obtained from http://www.bfro.net/

tall (or how short, depending on how you look at it) the subject of the Patterson film actually was.

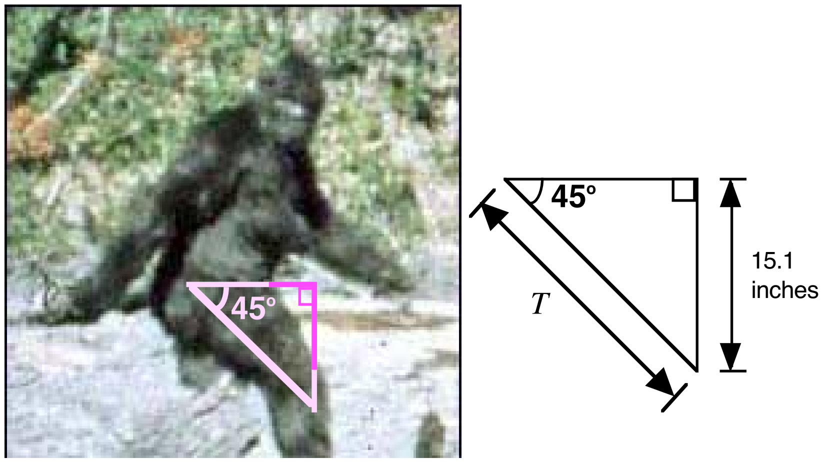

Figure 2: Creating a right angle triangle that can help you to calculate the diagonal length of the thigh, which is how much length the thigh could contribute to the subject's height if the subject stood erect.

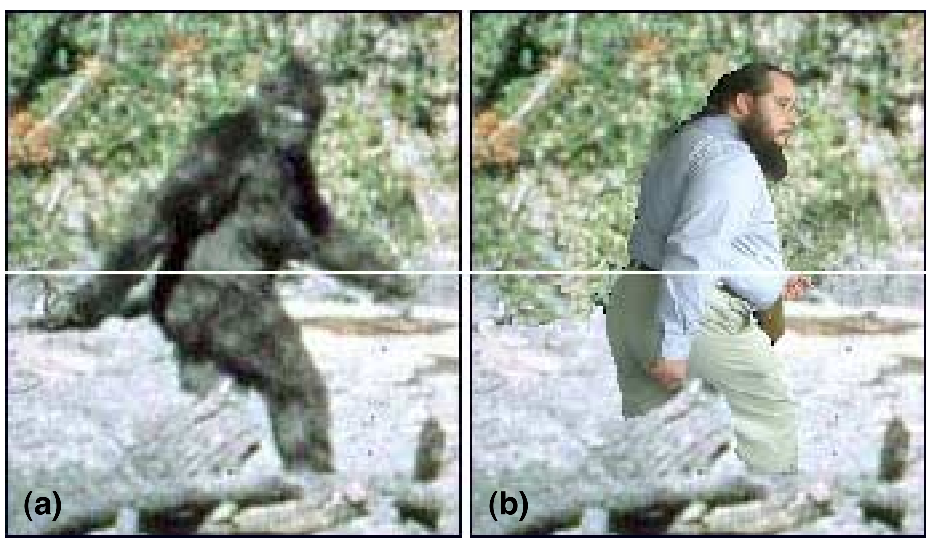

Figure 3: (a) Frame 352 from the Patterson film. (b) A human of similar height (approximately five feet ten inches tall) in a similar location and pose.

4. For ease of reference, Figure 6 from the homework assignment is reproduced here. Note that two angles (φ and ψ) have been added to the figure here. (These angles were not labeled in the version of Figure 6 that appeared in the homework assignment.)

Note that, from the triangle definitions of the trigonometric functions, the length of the side R and the angle ψ are related by the equation

$$\frac{\text{Opposite}}{\text{Adjacent}} = \frac{R}{16.5} = \tan(\psi).$$

Solving for R gives that: $R = 16.5 \cdot \tan(\psi)$.

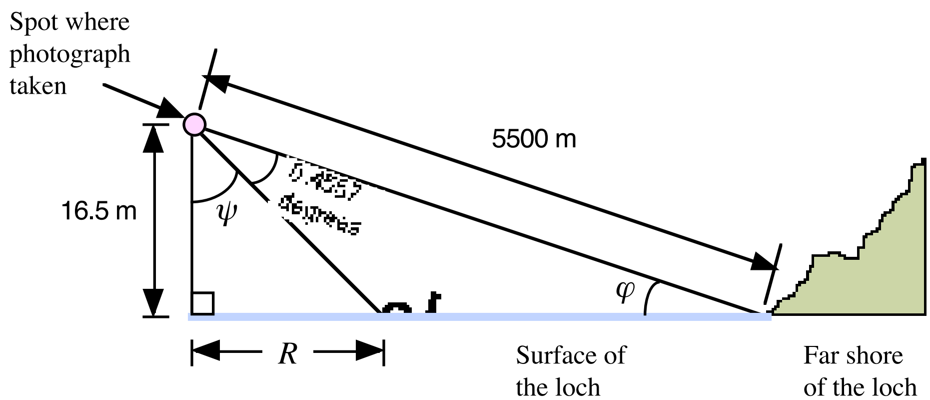

Figure 6: Diagram showing measurements made by Adrian Shine. The angle given is 0.4557 degrees.

So, if we can just find the value of the angle ψ then we will be able to calculate the distance on the surface of loch between the photographer and the object photographed.

To find the angle ψ, note that the internal angles of a triangle add to 180° or π radians. The internal angles of the large triangle shown in Figure 6 are (expressed in radians) φ, $\psi + \frac{\pi}{180} \cdot 0.4557$, and the right angle equal to $\frac{\pi}{2}$ radians. As the sum of these internal angles must be equal to π radians, we can write

$$\varphi + \psi + 0.0079534654 + \tfrac{\pi}{2} = \pi,$$

which is equivalent to:

$$\psi = 1.562842861 - \varphi.$$

To calculate the angle ψ, we need to find the value of the angle φ. Observe from the simplified version of Figure 6 (below) that:

$$\frac{\text{Opposite}}{\text{Hypoteneuse}} = \frac{16.5}{5500} = \sin(\varphi).$$

Using the inverse sine function gives: $\varphi = \sin^{-1}\left(\frac{16.5}{5500}\right) = 0.0030000045$ radians. Substituting this number into the equation for the angle ψ, we obtain:

$$\psi = 1.559842857 \text{ radians.}$$

Finally, substituting this value for the angle ψ into the equation

$$R = 16.5 \cdot \tan(\psi),$$

gives $R \approx 1506.31$ meters. Therefore, the distance across the surface of Loch Ness between Chris Rivett and Millissa Bavister and the object that they photographed was about 1500 meters, or slightly less than one mile[4]

5. The triangle shown in Figure 7 of the homework assignment is not a right angle triangle, so you cannot use trigonometric functions on it directly. Instead, we will have to break the triangle up into right triangles. Figure 7 (together with a way to break it down into right triangles) is shown below.

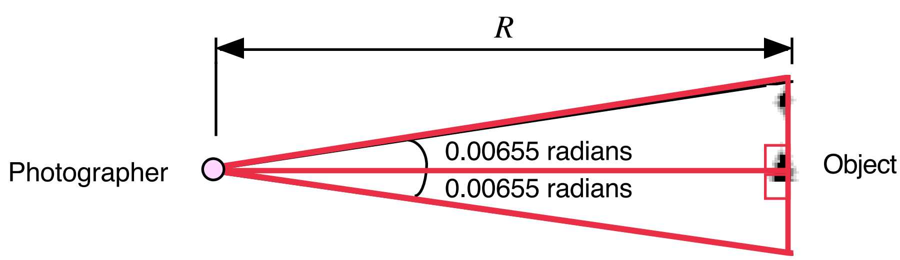

Figure 7: Overhead ("bird's eye") view of photographer and object on Loch.

So, this triangle can be drawn as two (identical) right angle triangles. The "opposite" side of each of these right angle triangles will be equal to one half of the length of the object. The length of the "adjacent" side of each right angle triangle is equal to R = 1506.31 calculated in Question 4. Using the definitions from triangle trigonometry, the length of each of these "opposite" sides is:

$$Opposite = (1506.31) \cdot \tan(0.00655) \approx 9.87 \text{ meters.}$$

The total length of the object will be twice this – that is, approximately 19.73 meters.

[4] One mile is approximately equivalent to 1610 meters.

Extra Credit

(a) Figure 4[5] (below) shows a version of the photograph with Kevin Anderson and his *Gigantopithecus* sculpture, but with some measurements, angles and right triangles superimposed over the original image.

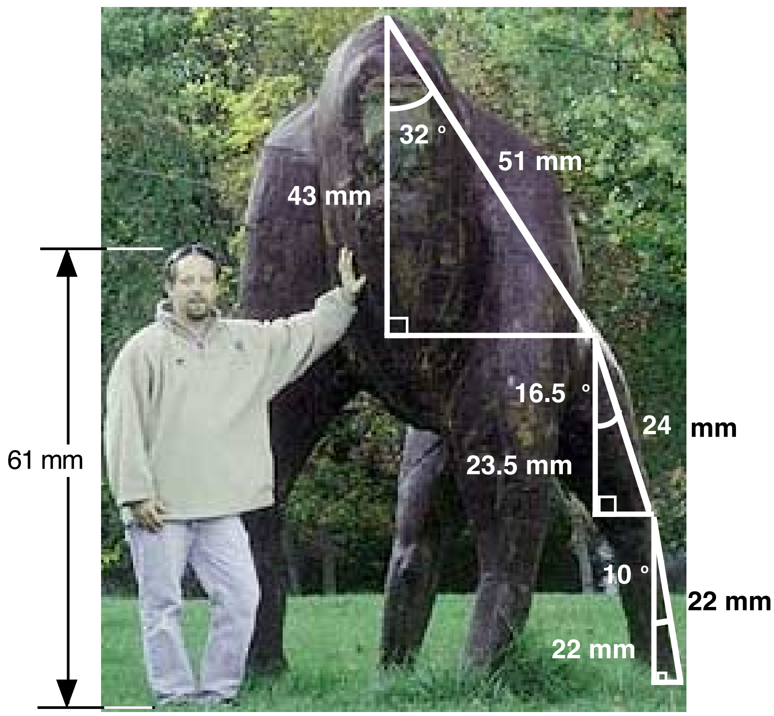

Figure 4: Artist Kevin Anderson standing next to the *Gigantopithecus* statue.

Kevin Anderson is approximately six feet tall. Therefore, 61mm on Figure 4 is equivalent to approximately 72 inches of actual height. Using this scaling factor (and the definitions of triangle trigonometry if you want to) you can determine the actual "diagonal" lengths of the upper body, thigh and shin of the *Gigantopithecus* sculpture. These results are summarized in Table 3 (below).

Body part	Approximate actual "diagonal" length (inches)
Upper body	60.2
Thigh	28.33
Shin	25.97

Table 3

[5] The source of the photograph of Kevin Anderson and the sculpture is: http://www.kandervision.com/homepix/giganto3-604c.jpg

Therefore, if Mr. Anderson's sculpture is an accurate representation of the physical size and posture of *Gigantopithecus*, this prehistoric giant ape would have been about (60.2 + 28.33 + 25.97) = 114.5 inches tall when standing fully erect. This is about nine feet six inches, which is about one and a half times the height of the subject in Roger Patterson's film.

(b) In Homework 18, the equation obtained for the daily food consumption of a fish-eating aquatic predator was:

$$g(L) = 3.2349 \cdot L + 3.5161$$

where L is the length of the animal's body (expressed in units of meters) and $g(L)$ is the daily food requirement for that animal (expressed in units of kilograms).

Substituting $L = 19.73$ into this equation gives a daily food consumption of

$g(L) = 67.34$ kg of fish per day for each animal of this length.

In Homework 18, we calculated that there was about 14200 kg of fish biomass available in Loch Ness. Likewise, in Homework 18 we noted[6] that a minimum of 10-20 such animals would likely be needed to maintain a viable breeding population. If the dark patch on the surface of Loch Ness photographed by Rivett and Bavister really is a fish-eating aquatic predator, then the smallest viable population (10) of such creatures would devour all of the fish in the Loch within:

$$\frac{14200}{673.4} \approx 21.09 \text{ days.}$$

This is just over three weeks, so it is difficult to see how the photograph taken by Rivett and Bavister could possibly be a fish-eating aquatic creature. There is simply not enough food available in Loch Ness to support a viable breeding population of such creatures for any substantial length of time.

[6] Source: R.W. Sheldon and S.R. Kerr. (1972) "Density of monsters in Loch Ness." *Limnology and Oceanography*, **17**: 746-798.