Homework Assignment 20: Due at the beginning of class 4/24/02.

The mathematical content of this homework assignment will bring together a lot of what you have learned about modeling phenomena with trigonometric functions, differentiating and locating the solutions of trigonometric equations.

Figure 1: Priest, professor, philosopher and economist Thomas Robert Malthus (1766-1834).

In Math Xa you completed a lab on the predictions made by nineteenth century philosopher and economist Thomas Malthus (see Figure 1[1]). In that lab you created a function that gave the annual agricultural production of the United States as a function of time (see Figure 2[2]).

When you modeled the agricultural production of the United States in the Malthus lab, you were encouraged to use a linear function. This approach gave a function for food production that agreed with Malthus' prediction[3]:

"…population increases in a geometric ratio, while the means of subsistence increases in an arithmetic ratio."

The overall trend in Figure 2 is certainly an increasing trend, but a simple linear function is not a very accurate representation of the curve in Figure 2. For example, a simple linear function cannot represent the obvious oscillations in Figure 2.

In Questions 1, 2 and 3 of this homework assignment, you will use trigonometry to create a much more accurate model of agricultural production that captures both the increasing trend and the oscillations in agricultural production.

In Questions 4 and 5 of this homework assignment you will use the function that you have created along with the calculus of sine and cosine that you have learned to do some of the planning calculations that governments and non-governmental organizations (NGO's) perform.

---

[1] Image source: http://www.bluepete.com/Literature/Biographies/Philosophy/Malthus.htm

[2] The data used to construct Figure 2 were obtained from: U.S. Department of Agriculture, Foreign Agricultural Service. *Grain: World Markets and Trade*. October, 2000. Available on-line from: http://www.fas.usda.gov/grain_arc.html

[3] Source: T.R. Malthus. *An Essay on the Principle of Population.* New York: Penguin Books, 1985.

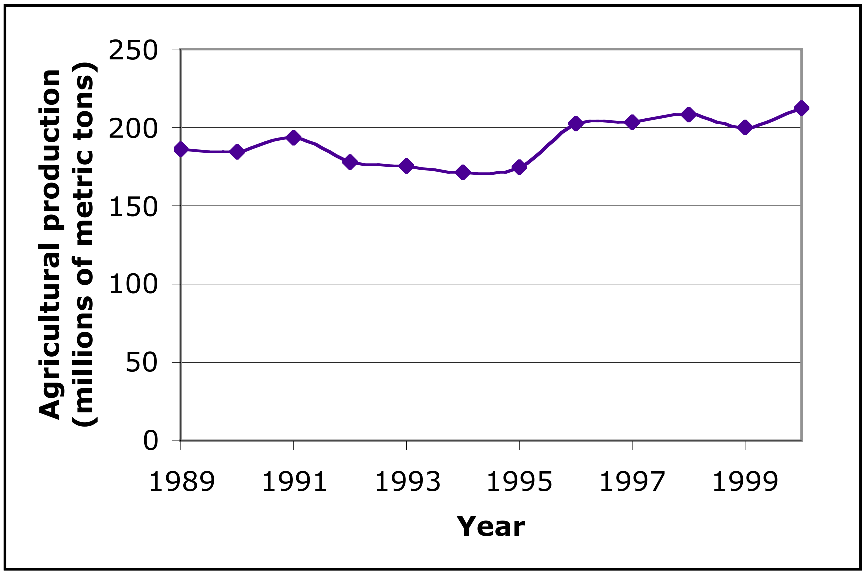

Figure 2: Annual U.S. Agricultural Production, 1989-2000.

1. Table 1 gives the annual agricultural production[4] of the United States from 1989 to 2000 in units of "millions of metric tons of agricultural produce." The quantity *T* represents the number of years since 1989.

Year	*T*	Agricultural production (millions of metric tons)
1989	0	186.2
1990	1	184.7
1991	2	194.0
1992	3	178.2
1993	4	175.4
1994	5	171.2
1995	6	174.6
1996	7	202.3
1997	8	203.6
1998	9	208.6
1999	10	199.9
2000	11	212.7

Table 1: Annual U.S. Agricultural Production, 1989-2000.

Enter the data from Table 1 into a graphing calculator to create a function $L(T)$. This function is the linear function that you created while working on the Thomas Malthus lab in Math Xa. Sketch a graph showing the data from Table 1 and the function $L(T)$. (Axes for this may be downloaded separately.)

---

[4] Source: U.S. Department of Agriculture, Foreign Agricultural Service. *Grain: World Markets and Trade*. October, 2000. Available on-line from: http://www.fas.usda.gov/grain_arc.html

2. In this question, the idea is to create a numerical and graphical representations of the difference between the actual level of agricultural production and the value given by the linear function $L(T)$. In Question 3 you will use these numerical and graphical representations to create an equation for a periodic function.

Use the function $L(T)$ that you created in Question 1 to complete Table 2 below.

Year	T	Agricultural Production	$L(T)$	Agricultural Production Minus $L(T)$
1989	0	186.2		
1990	1	184.7		
1991	2	194.0		
1992	3	178.2		
1993	4	175.4		
1994	5	171.2		
1995	6	174.6		
1996	7	202.3		
1997	8	203.6		
1998	9	208.6		
1999	10	199.9		
2000	11	212.7		

Table 2.

Plot the numbers from the last column of Table 2 against the values of T. (Axes for this may be downloaded separately.)

3. Which of the following functional forms

- $f(x) = A \cdot \sin\left(\frac{2\pi}{P} \cdot x\right) + M$
- $f(x) = -A \cdot \sin\left(\frac{2\pi}{P} \cdot x\right) + M$
- $f(x) = A \cdot \cos\left(\frac{2\pi}{P} \cdot x\right) + M$
- $f(x) = -A \cdot \cos\left(\frac{2\pi}{P} \cdot x\right) + M$

will do the best job of representing the graph that you drew in Question 2? Briefly (in a sentence or two) explain your reasoning. Find an equation for graph that you drew in Question 2, and combine this equation with the linear function $L(T)$ to create an equation for the annual agricultural production of the United States. Once you have created the equation for the annual agricultural production of the United States, plot a graph showing the data from Table 1 and the graph of the function that you have created. (Axes for this may be downloaded separately.)

In the Thomas Malthus lab, you calculated that the per capita food production of the United States was currently well above the level necessary for basic human survival, and that even if the population of the United States grew exponentially, the per capita food production would remain well above basic survival well into the 24th century. Not every country in the world is in such a fortunate situation. For example, many African are currently unable to produce enough food to meet even the base survival needs of their population, or will soon be unable to meet the survival needs of their population. The U.S. Department of Agriculture estimates that by 2010 (eight years away) more than 60% of the 37 countries that make up sub-Saharan Africa (see Figure 3[5] will be unable to meet the basic survival needs of their population[6].

In the Biblical story of Genesis 41:1-37 (you can download this separately and read it if you wish to), the Pharaoh of Egypt has a prophetic dream in which he sees seven well-fed cattle, followed by seven "lean-fleshed and ill-favored" cattle which consumed the well-fed cattle. The Pharaoh's butler has a Hebrew servant named Joseph who interprets the dream to mean that Egypt will enjoy seven years of abundant harvests followed by seven years of drought. Joseph advises the Pharaoh to appoint overseers to collect 20% of the grain harvested in the years of plenty, and to store this surplus to prevent a famine in the seven years of drought.

---

[5] Image source: http://www.nbii.gov/

[6] Source: U.S. Department of Agriculture, Economic Research Service. "Global Food Security Overview." Food Security Assessment GFA-12, December 2000. You can also look up a calculation that you did for the sub-Saharan nation of Kenya in Math Xa Homework Assignment #20, (see http://www.courses.fas.harvard.edu/~mathxa/homework/assignments/xa_hw20.pdf and http://www.courses.fas.harvard.edu/~mathxa/homework/assignments/xa_hw20_sol.pdf) in which you calculated that by 2007-2008 (five or six years away) Kenya will no longer be able to produce enough maize (by far the most important food crop) for its entire population.

The plan that Joseph suggested to Pharaoh is a simple idea – an idea practiced by such animals as the Black-capped chickadee (*Poecile atricapilus*) – that excess food should be stored in times of plenty in a food cache and retrieved in times of need. The difficult part – as governments and humanitarian relief organizations generally do not have prophetic dreams on which to base their long-term plans – is predicting when the times of plenty and the times of need are likely to occur. In Questions 4 and 5 you will use calculus and the function that you created in Questions 1, 2 and 3 to do this.

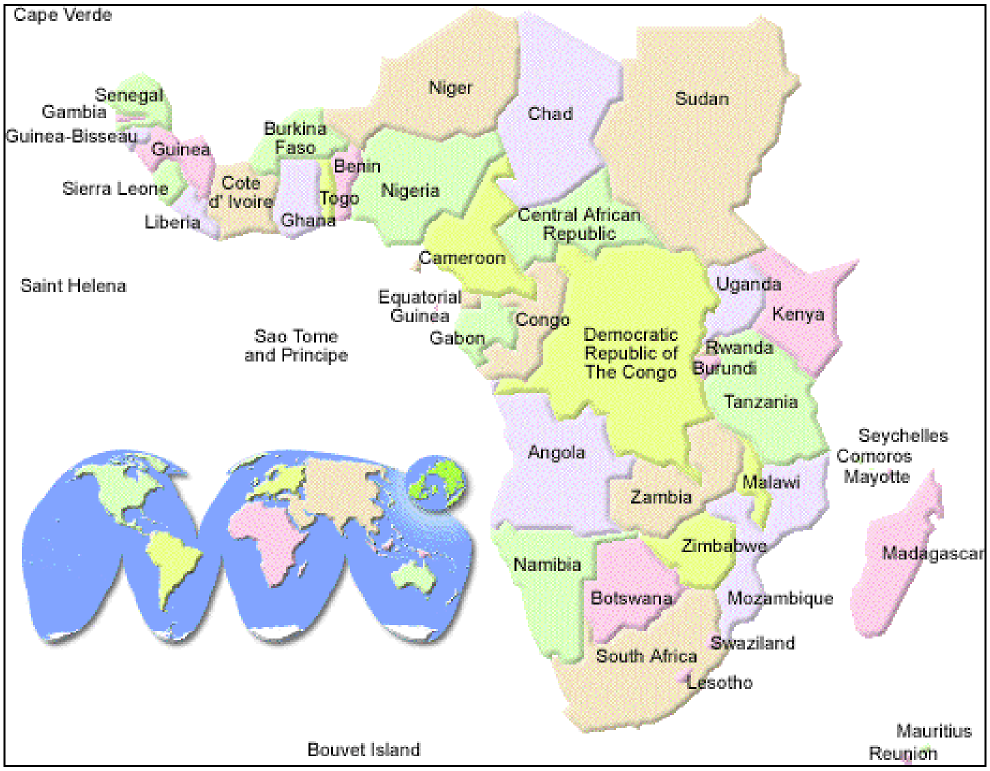

Figure 3: The nations of sub-Saharan Africa. This collection of nations represents the majority of African nations.

4. Find equations for the first and second derivatives of the function that you created in Question 3. Use the equation for the first derivative to locate *all* critical points (there are infinitely many of them) of the function that you created in Question 3.

5. Use the equations that you found in Question 4 to determine when the agricultural production of the United States will reach:

- A local maximum, or,
- A local minimum,

and indicate which years give a local maximum, and which years give a local minimum of agricultural production.

NOTE: **1.** You need to find and describe *all* years when the agricultural production of the United States has a local maximum and *all* years when the agricultural production of the United States has a local minimum. There are infinitely many such points, so don't try writing them all out one-by-one.

2. You need to provide some evidence to show that the points that you have identified really are local maximums and local minimums. Furthermore, this evidence should establish that *all* of the points identified are local maximums or local minimums. That is, don't just give explicit calculations for one or two points and assume that's enough.