Homework Assignment 22: Due at the beginning of class 4/29/02.

This homework assignment is intended to be reasonably straightforward. If you find that you can solve the problems easily and with little fuss, then that's a good sign rather than a sign that you have missed the point.

One of the difficulties that many people have when they first learn about the antidifferentiation formulas:

- $\int \frac{1}{\sqrt{1-x^2}} dx = \sin^{-1}(x) + C$
- $\int \frac{-1}{\sqrt{1-x^2}} dx = \cos^{-1}(x) + C$
- $\int \frac{1}{1+x^2} dx = \tan^{-1}(x) + C$

is recognizing when it is appropriate to use them. The point of Questions 1-3 is to help you learn to recognize when to use the antiderivative formulas involving the inverse trigonometric functions. In each question you are presented with a pair integrals that seem to be very similar. One of the integrals in the pair will require the antidifferentiation formulas listed above, the other integral in the pair will not.

1. Find equations for the two indefinite integrals given below. Your equations should contain one unspecified constant.

$$\int \frac{2}{\sqrt{1-(2x)^2}} \cdot dx \qquad \text{and} \qquad \int \frac{2x}{\sqrt{1-(2x)^2}} \cdot dx.$$

2. Find equations for the two indefinite integrals given below. Your equations should contain one unspecified constant.

$$\int \frac{4x^3}{1+x^8} \cdot dx \quad \text{and} \quad \int \frac{4x^7}{1+x^8} \cdot dx.$$

3. Find equations for the two indefinite integrals given below. Your equations should contain one unspecified constant.

$$\int \frac{1}{\sqrt{1-x}} \cdot dx \quad \text{and} \quad \int \frac{1}{\sqrt{x}} \cdot \frac{1}{\sqrt{1-x}} \cdot dx.$$

Hint: Try $u = \sqrt{x}$.

Many creatures exhibit a behavior called an *escape response*. When frightened, the creature will accelerate very suddenly and dart away to avoid some perceived threat. A classic example of an animal escape response is exhibited by many species of squid which use their powerful siphons to suddenly "jet" away from threats. As an additional form of protection, squid often release a cloud of camouflaging ink as they escape. One of the visual cues[1] thought to stimulate an escape response is the rate of change of the *apparent angle* of an approaching object.

The diagram[2] given below shows an object approaching a squirrel (*Sciurus carolinensis*). The object is moving and the squirrel is sitting still. The distance between the squirrel and the object is represented by the function $f(T)$. The apparent angle of the object (as seen by the squirrel) is represented by the function $a(T)$. T represents the time (in seconds) since the squirrel first spotted the approaching object.
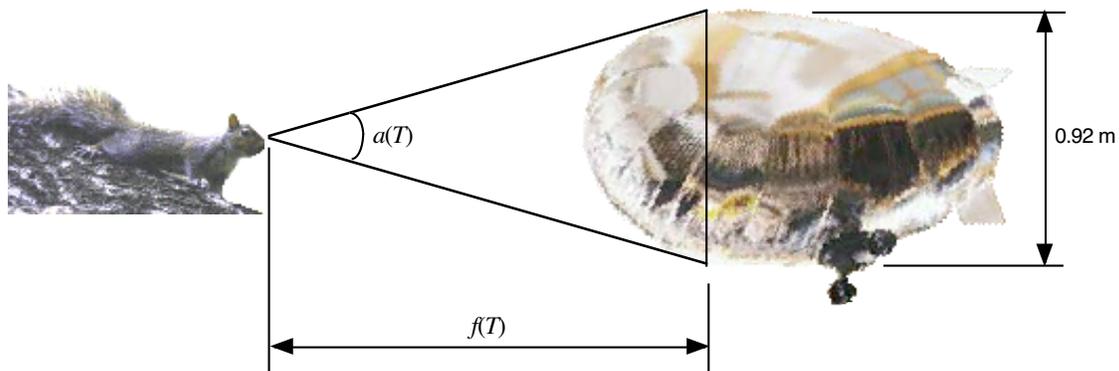
4. The diameter of the object is 0.92 meters. Find an equation for the apparent angle, $a(T)$. Your equation may include the function $f(T)$.

5. The object travels with a speed[3] of 0.38 meters per second. How quickly is the apparent angle changing when the object is 3 meters from the squirrel? (Your answer should be given in units of radians per second.)

**Extra Credit Opportunity: Up to 10 points available.**

In class on Friday you probably learned the two antidifferentiation formulas:

- $\int \frac{1}{\sqrt{1-x^2}} dx = \sin^{-1}(x) + C$
- $\int \frac{-1}{\sqrt{1-x^2}} dx = \cos^{-1}(x) + C$

There is a second way to calculate the antiderivative $\int \frac{-1}{\sqrt{1-x^2}} \cdot dx$. This second method of calculation is shown below.

$$\int \frac{-1}{\sqrt{1-x^2}} \cdot dx = (-1) \cdot \int \frac{1}{\sqrt{1-x^2}} \cdot dx = (-1) \cdot \sin^{-1}(x) + C = -\sin^{-1}(x) + C.$$

Does this mean that $\cos^{-1}(x) = -\sin^{-1}(x)$? Before you commit yourself to an answer, consider the graphs of $y = \sin^{-1}(x)$ and $y = \cos^{-1}(x)$ (see below).
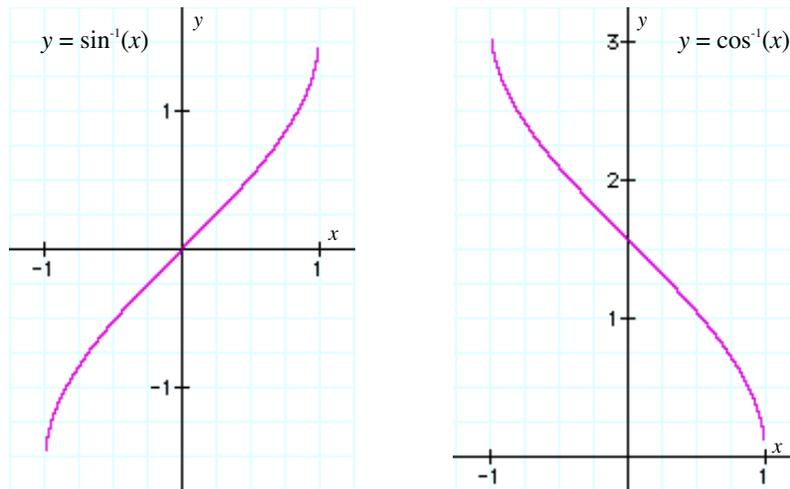
To answer this question you should either mathematically demonstrate that:

$$\cos^{-1}(x) = -\sin^{-1}(x),$$

or, if you do not believe that this is the case, explain how it is possible for both of the antidifferentiation rules:

- $\int \frac{-1}{\sqrt{1-x^2}} dx = -\sin^{-1}(x) + C$
- $\int \frac{-1}{\sqrt{1-x^2}} dx = \cos^{-1}(x) + C$

to be correct even though $\cos^{-1}(x) \neq -\sin^{-1}(x)$.