Homework Assignment 3: Due at the beginning of class 2/11/02

1.   If we let T represent the number of years since 1990 and $N(T)$ represent the number of high school seniors who have used some illicit drug, then the derivative is given by the equation[1]:

$$N'(T) = -0.65 \cdot \big(N(T) - 43,200 \big).$$

Determine which of the following functions are solutions of this differential equation? Remember to provide evidence to back up your conclusions.

(I) $N(T) = 43200 - 26434.46 \cdot e^{-0.65T}$.

(II) $N(T) = 29400 \cdot \sqrt{T}$.

(III) $N(T) = 43200 - 13800 \cdot e^{-0.65T}$.

2.   In 1991, the number[2] of high school seniors who had used any kind of illicit drug[3] was about 29,400[4]. Which of the functions given in Question 1 are consistent with this information? Remember to provide evidence to back up your conclusions.

Figure 1: Insulated commuter cup tested as best by the Hamaacher-Schlemmer institute.

Newton's Law of Cooling is an example of a differential equation that you have had some experience with in Math Xa. In Question 3 you will finally learn the formula for the function that gives the temperature of the infamous Hammacher-Schlemmer cup (see Figure 1[5]).   In this situation, boiling water (temperature = 100°C) has been poured into the Hammacher-Schlemmer cup. Let t represent the amount of time that has elapsed (in minutes) and $T(t)$ the temperature of the water in degrees Celsius.

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[1] The equation for the derivative is obtained from data recorded in the Monitoring the Future Study, 2001.

[2] Remember (from assignment 1) that here the "number of illicit drug users" means: If you took a random sample of 100,000 high school seniors from across the United States, the number of seniors in this group who had used illicit drugs at some point in their lives would be the "number of illicit drug users."

[3] This includes marijuana/hashish, inhalants, nitrites, LSD, hallucinogens other than LSD, PCP, Ecstasy, Cocaine (powder and crack), heroin, amphetamines, barbiturates, tranquilizers, rohypnol, GHB, ketamine, but **excludes** alcohol, tobacco and steroids.

[4] Source: US Department of Health and Human Services, National Institute on Drug Abuse, Monitoring the Future Study, 2001.

[5] Image source: http://www.hammacher.com/

3. The version of Newton's Law of Cooling that was appropriate for the Hammacher-Schlemmer cup was:

$$T'(t) = -0.0033 \cdot [T(t) - 20].$$

Show that the function:

$$T(t) = 20 + 80 \cdot e^{-0.0033t}$$

is a solution of Newton's Law of Cooling, and that this function predicts the correct initial temperature of the water.

4. Locate any equilibrium solutions of Newton's Law of Cooling:

$$T'(t) = -0.0033 \cdot [T(t) - 20].$$

and classify the equilibrium solutions as stable, unstable or semi-stable. Explain what these equilibrium solutions represent in terms of the temperature of the water in the Hammacher-Schlemmer cup, and why it makes sense (in terms of how objects heat up and cool down) for these equilibrium solutions to be stable, unstable or semi-stable.

5. In Question 3, the differential equation was:

$$T'(t) = -0.0033 \cdot [T(t) - 20]$$

and the initial condition was:

$$T(0) = 100.$$

The equation for the function $T(t)$ that is a solution of the differential equation and which also satisfies the initial condition is:

$$T(t) = 20 + 80 \cdot e^{-0.0033t}.$$

Try to notice a pattern in how all of the numbers in the differential equation and the initial condition contribute to the equation for the function $T(t)$. Once you think you see the pattern, apply it to find an equation for the function $P(t)$ where:

$$P'(t) = 0.01 \cdot [P(t) - 400]$$

$$P(0) = 100.$$

As part of your answer, you should show that the equation that you find for $P(t)$ is a solution of the differential equation, and that it agrees with the initial condition.