Homework Assignment 5: Solutions

1. The probabilities that the naval aviator from VA-145 will land on any particular attempt are given in Table 1 below. Please note that I have typed these numbers out (rather than just writing the decimal equivalents) so that it will be easier for you to see how these numbers were obtained, and also how these numbers are used in Question 2.

Attempt at landing	Probability that VA-145 pilot will land successfully on that attempt:
First	0.98
Second	$0.98\cdot(0.02)$
Third	$0.98\cdot(0.02)^2$
Fourth	$0.98\cdot(0.02)^3$
Fifth	$0.98\cdot(0.02)^4$

Table 1.

2. The probabilities that the naval aviator from VA-145 will land on one of several attempts are given below. Please note that I have typed these out as sums of numbers to make it clearer how these results are related to geometric series. If you actually evaluated these probabilities and just wrote down the results, that is perfectly acceptable.

VA-145 pilot lands successfully on:	Probability that this will happen
First attempt	0.98
First or second attempt	$0.98 + 0.98\cdot(0.02)$
First, second or third attempt	$0.98 + 0.98\cdot(0.02) + 0.98\cdot(0.02)^2$
First, second, third or fourth attempt	$0.98 + 0.98\cdot(0.02) + 0.98\cdot(0.02)^2 + 0.98\cdot(0.02)^3$
First, second, third, fourth or fifth attempt	$0.98 + 0.98\cdot(0.02) + 0.98\cdot(0.02)^2 + 0.98\cdot(0.02)^3 + 0.98\cdot(0.02)^4$

Table 2.

3. If you look at the entries in Table 2, then you may be able to discern a pattern. Each of the terms has a factor of 0.98, and a factor of 0.02 which has been raised to a power. The probability that an aviator from VA-145 will land sometime during his or her first n attempts will be given by:

$$Probability \;=\; 0.98 + 0.98\cdot(0.02) + 0.98\cdot(0.02)^2 + \ldots + 0.98\cdot(0.02)^{n-2} + 0.98\cdot(0.02)^{n-1}.$$

A sum of the form $a + a\cdot r + a\cdot r^2 + \ldots + a\cdot r^q$ is equal to $\frac{a\cdot\left(1-r^{q+1}\right)}{1-r}$. Using $a = 0.98$ and $r = 0.02$, the probability written out above would sum to:

$$Probability \;=\; \frac{0.98\cdot\left(1-0.02^n\right)}{1-0.02} = 1-0.02^n.$$

As $n \to \infty$, the probability will approach a limit of 1. This is because $(0.02)^n$ gets smaller and smaller, approaching a limit of zero as $n \to \infty$.

A probability of 1 represents that the event will happen with total certainty. Hence, if no limit is placed on the number of attempts that an aviator from VA-145 is allowed to make at landing, it is certain that he or she will eventually land.

4. The expected number of attempts at landing that a minimally competent naval aviator would have to make is expressed as the infinitely long string of terms given below. Note that the string of terms written down here includes a "general term" (i.e. n*$0.75\cdot(0.25)^{n-1}$) to establish the pattern and ends with the ellipsis "..." signifying that the terms continue on forever following the general pattern indicated.

$$\textit{Expected number of attempts to land} \;=\; 1*0.75 + 2*0.75\cdot(0.25) + 3*0.75\cdot(0.25)^2 + 4*0.75\cdot(0.25)^3 + 5*0.75\cdot(0.25)^4 + \ldots + n*0.75\cdot(0.25)^{n-1} + \ldots$$

5. The series written down as the answer to Question 4 is **not** a geometric series. One of the defining characteristics of a geometric series is that the ratio of adjacent terms in the series is always the same (and equal to the multiplicative factor r).

For the series written down for Question 4, the ratio of the second term over the first term is:

$$Ratio \;=\; \frac{2*0.75\cdot(0.25)}{1*0.75} \;=\; 0.5.$$

However, the ration of the third term over the second term is:

$$Ratio \;=\; \frac{3*0.75\cdot(0.25)^2}{2*0.75\cdot(0.25)} \;=\; 0.375.$$

As these two ratios are not the same, the series written down for the expected value in Question 4 cannot be a geometric series.