Homework Assignment 6: Due at the beginning of class 2/20/02

The largest purchase that most Americans will ever make is their house. Most people do not have the financial resources to buy their houses outright and instead take out a mortgage.

In the United States, the interest on mortgages is compounded monthly and payments are made monthly. In the United States, mortgages are usually paid off over a 15 year or 30 year period. Often when trying to secure a loan for a property, you will have to produce 10% of the cost of the property up front. You take out a loan for the remaining 90% of the cost of the property and pay this back to the lender in the form of monthly payments over the term of the loan.

Last summer, a condominium[1] was advertised in a nice community called "The Falls at Faxon Park." This new community is located in Quincy, MA, close to stores, schools and the MBTA Red Line. The asking price for the condominium was $179,000[2]. Inquiries at a local bank[3] indicated that at the time, the fixed interest rate mortgages were charging about 8.0% per annum.

If you were in the market for a property, and the condominium described above sounded interesting, then the first step in deciding whether or not to buy would be to estimate whether or not you could afford the monthly payments on the property. In Questions 1 and 2 you will make rough estimates on the size of the monthly payments.

1.   A thirty year mortgage is the most affordable, as it has the lowest monthly payments. If you have enough to cover 10% of the purchase price of the condo, about how much do you have to borrow? If all that you had to pay was the amount that you borrowed (i.e. no interest) how much would your monthly payment be? This figure will serve as an under-estimate for the actual monthly payment.

---

[1] Image source: http://www.rent.net
[2] Source: Century 21 Liberty Associates, 1212 Hancock St., Quincy MA 02169.
[3] Source: Citizens Bank Mortgage Center, 1200 Hancock St., Quincy MA 02169.

2. If you made no payments on this loan during the 30 year period, how much would you owe by the end of 30 years? If you divide this total by 360 (12 months times 30 years) you will get an over-estimate for the monthly mortgage payment. Calculate this over-estimate for the actual monthly mortgage payment.

In Questions 3, 4 and 5 you will make careful calculations using geometric series to work out the exact monthly payments that you would have to make to the bank in order to pay off the mortgage. All of the information (purchase price, interest rates, etc.) given at the beginning of this homework assignment are relevant to Questions 3-5. Assume that you will take out a 30 year mortgage.

3. Remember that a bank will compound the interest that you owe before deducting your payment from the amount that you still own them. In this problem, use the symbol M to denote your monthly payment in dollars. Find an expression that gives the size of the outstanding balance of the mortgage after 1 year (12 months).

4. Remember that a bank will compound the interest that you owe before deducting your payment from the amount that you still own them. In this problem, use the symbol M to denote your monthly payment in dollars. Find an expression that gives the size of the outstanding balance of the mortgage after 30 years (360 months).

5. Set the expression that you obtained in Question 4 equal to zero and use what you have learned about geometric series to calculate the monthly payment (M) for a 30 year mortgage.

Double-Check: In Questions 1 and 2 you calculated an under-estimate and an over-estimate for the actual monthly payment. Your answer to Question 5 should fall somewhere between your answers to Questions 1 and 2. If this is not the case, go back and check your calculations.