

Homework Assignment 9: Due at the beginning of class 2/27/02

In Homework Assignment 5, you used series to calculate the probabilities of landing on an aircraft carrier. Questions 1 and 2 of this homework assignment will continue your analysis of carrier landings.

In Homework 5, the “boarding rate” of a pilot was defined to be the probability that a pilot will land on an aircraft carrier. For example, a boarding rate of 0.9 means that a pilot will successfully land on 9 out of every 10 attempts. According to US Navy regulations¹, pilots must achieve a daytime² boarding rate of 0.75 in order to be qualified to operate from an aircraft carrier.

In Homework 5 you were asked to find an infinite series that gave the “expected value” for the number of attempts that a minimally competent carrier pilot would make. The series representing this expected value is:

$$\begin{aligned} \text{Expected number} &= 1 \cdot 0.75 + 2 \cdot 0.75 \cdot (0.25) + 3 \cdot 0.75 \cdot (0.25)^2 \\ \text{of attempts to land} &+ 4 \cdot 0.75 \cdot (0.25)^3 + 5 \cdot 0.75 \cdot (0.25)^4 + \\ &+ \dots + n \cdot 0.75 \cdot (0.25)^{n-1} + \dots \end{aligned}$$

Written in sigma notation, this expected value could be represented as:

$$\text{Expected value} = \sum_{k=1}^{\infty} k \cdot (0.75) \cdot (0.25)^{k-1}$$

The expected value of a quantity is often interpreted as an average. For example in Homework 5, the expected value of the total of two dice was calculated and found to be equal to 7. Suppose that you threw the dice many times and recorded the total that you got each time. If you were to average all of the numbers that you had recorded, then you would get an average that was very close to 7.

1. How could you interpret the expected value of the number of attempts to land that a naval aviator has to make? Describe your interpretation in a few sentences. Given your interpretation of the expected value, do you expect the infinite series:

$$\sum_{k=1}^{\infty} k \cdot (0.75) \cdot (0.25)^{k-1}$$

to converge or diverge? Explain your reasoning in a sentence or two.

¹ COMNAVAIRPAC Instruction 3740.2W and COMNAVAIRLANT Instruction 3740.120, Section 9 Paragraph (c).

² The US Navy also conducts carrier operations during night. According to US Navy regulations, a pilot must have a night boarding rate of 0.65 in order to be qualified to operate from an aircraft carrier.

2. Use mathematics to determine whether the infinite series:

$$\sum_{k=1}^{\infty} k \cdot (0.75) \cdot (0.25)^{k-1}$$

diverges or converges.

NOTE: When you write out your answer for Question 2, you should include full details of any calculations that you do or reasoning that you employ. If you simply write “CONVERGES” or “DIVERGES” you won’t get full credit, even if your answer happens to be correct.

In Questions 3, 4 and 5 you are given a series that is represented using sigma (or summation) notation. In each case, you should determine whether the given series converges or diverges.

3.
$$\sum_{k=1}^{\infty} \frac{1}{k!}$$

NOTE: The symbol $k!$ represents the product of all of the positive integers from 1 up to k . That is:

$$k! = 1 \cdot 2 \cdot 3 \cdot \dots \cdot (k-1) \cdot k$$

4.
$$\sum_{k=1}^{\infty} \frac{(1.01)^k}{1+k^2}$$

5.
$$\sum_{k=1}^{\infty} \frac{k}{2 \cdot k + 1}$$

NOTE: When you write out your answer for Questions 3, 4 and 5 you should include full details of any calculations that you do or reasoning that you employ. If you simply write “CONVERGES” or “DIVERGES” you won’t get full credit, even if your answer happens to be correct.