



ICE - Slicing Problems

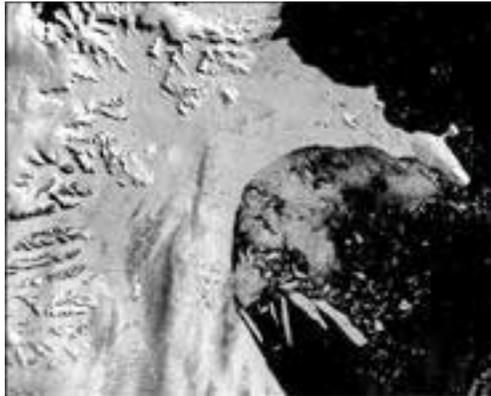


Figure 1: Satellite photograph of the Larsen B ice shelf, Antarctica. This photograph was made on February 23, 2002.

On March 18, 2002, the lead story in the New York Times concerned the break up of the Larsen B ice shelf (see Figure 1¹). In this spectacular event, an area of ice the size of Rhode Island splintered into pieces and fell into the ocean. Approximately 720 billion tons of ice (that had been stable for at least the last 50 years) broke apart. Scientists were stunned by the speed with which the ice shelf broke up (3-4 weeks)². Dr. David Vaughn (a glaciologist at Cambridge University in England) summarized the reaction

of the scientific community:

“We knew that what was left³ would collapse eventually, but the speed of it is staggering. It is hard to believe that 500 billion tons of ice sheet has disintegrated in less than a month.”

Last week⁴, scientists warned that more ice break-ups probably lie ahead.

Tim Naish, a New Zealand climate researcher based at the Institute of Geological and Nuclear Physics in Wellington New Zealand, has made a lengthy study of the Ross ice shelf (the largest in the Antarctic). The Ross ice shelf covers an area of approximately 332,000 square miles which is roughly the same size as France. A study released last year⁵ Dr. Naish and his collaborators suggested that the Ross ice shelf is now vulnerable to collapse. Computer predictions indicate that the collapse of the Ross ice shelf could lead to rises in sea levels of 15-20 feet worldwide.

¹ Image source: <http://www.nasa.gov/>

² Source: http://news.bbc.co.uk/1/hi/english/sci/tech/newsid_1880000/1880566.stm

³ The Larsen ice shelf consists of three main parts, prosaically named “Larsen A,” “Larsen B” and “Larsen C.” The Larsen A ice shelf broke up (much more slowly) in 1995. At the time of writing the Larsen C ice shelf was still intact.

⁴ Source: Amanda Onion. “Scientists warn Antarctica’s ice shelves face further break-ups.” *ABC News*, March 26, 2002.

⁵ Source: T.R. Naish et al. (2001) “Orbitally induced oscillations in the east Antarctic ice sheet at the Oligocene/Miocene boundary.” *Nature*, **413**(): 719-723.

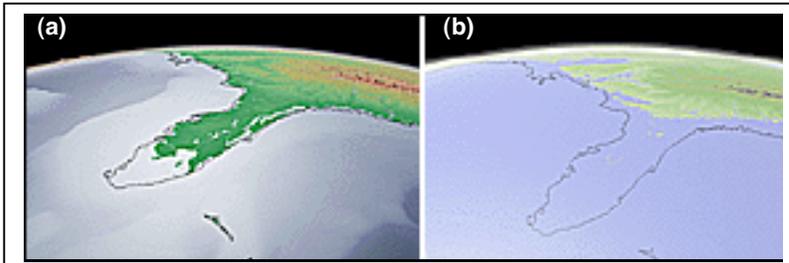


Figure 2: Computer predictions of results of global warming. (a) Changes to the Florida coastline as a result of the West Antarctic ice sheet melting and causing a 17 foot rise in sea levels. (b) Changes to the Florida coastline as a result of the East Antarctic ice sheet melting, causing a 170 foot rise in sea levels.

Figure 2(a)⁶ shows the predicted effects of a 17 foot rise in sea levels on the coastline of the United States. Figure 2(b) shows the effects of a much more severe 170 foot rise in sea levels that would result if all of the Antarctic ice sheets disintegrated.

Scientists have attributed the rapid break-up of the Larsen B ice sheet to rises in average world temperature – a phenomenon popularly called “global warming.” As Dr. Ted Scambos (a glaciologist at the National Snow and Ice Data Center in) noted⁷:

“We just saw how a small amount of warming that doesn’t seem like a big deal can cause billions of tons of ice to fragment and disappear. Now we know there’s potential for break-ups to happen rapidly.”

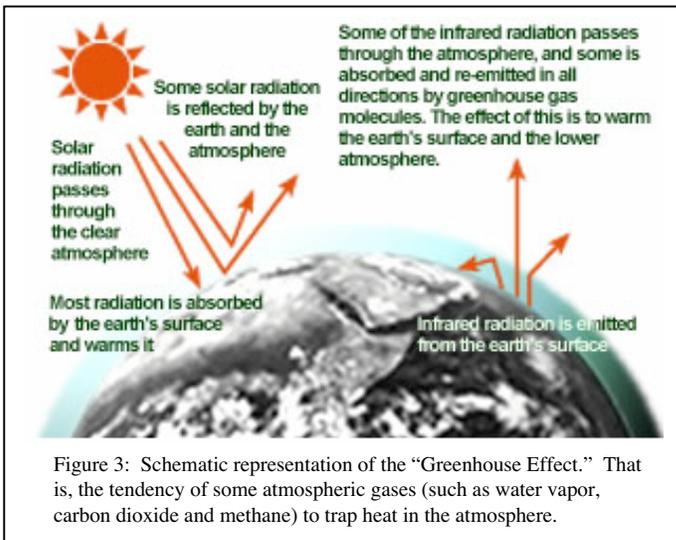


Figure 3: Schematic representation of the “Greenhouse Effect.” That is, the tendency of some atmospheric gases (such as water vapor, carbon dioxide and methane) to trap heat in the atmosphere.

Many credible scientists⁸ believe that the current phenomenon of global warming is either a direct result (or has been heavily influenced by) of human activity such as the large-scale release of so-called greenhouse gases⁹ into the atmosphere as a by-product of human industrial and agricultural activity. Figure

⁶ Image source: <http://www.pbs.org/wgbh/nova/warnings/waterworld/>

⁷ Source: Amanda Onion. “Scientists warn Antarctica’s ice shelves face further break-ups.” *ABC News*, March 26, 2002.

⁸ For example, see: J.T. Houghton, Y. Ding, D.J. Griggs, M. Noguer, P.J. van der Linden, X. Dai, K. Maskell and C.A. Johnson (eds.) *Climate Change 2001: The Scientific Basis*. Cambridge, England: Cambridge University Press, 2002.

⁹ The two most important greenhouse gases that are released by human activity (*anthrogenic* greenhouse gases) are carbon dioxide (CO₂) and methane (CH₄).

3¹⁰ shows the theory that underlies this position – that these anthropogenic greenhouse gases trap solar energy in the atmosphere leading to a warming of the surface of the Earth and the lower portions of the atmosphere (in particular the troposphere).

According to the EPA¹¹, the most important anthropogenic greenhouse gas is carbon dioxide (CO₂). Figure 4¹² shows the average concentration of carbon dioxide in the atmosphere from year 1000 of the common era to the present day. One of the clear messages from Figure 4 appears to be that since the beginning of the industrial revolution in Europe during the 19th century, the concentration of carbon dioxide in the Earth's atmosphere has risen dramatically.

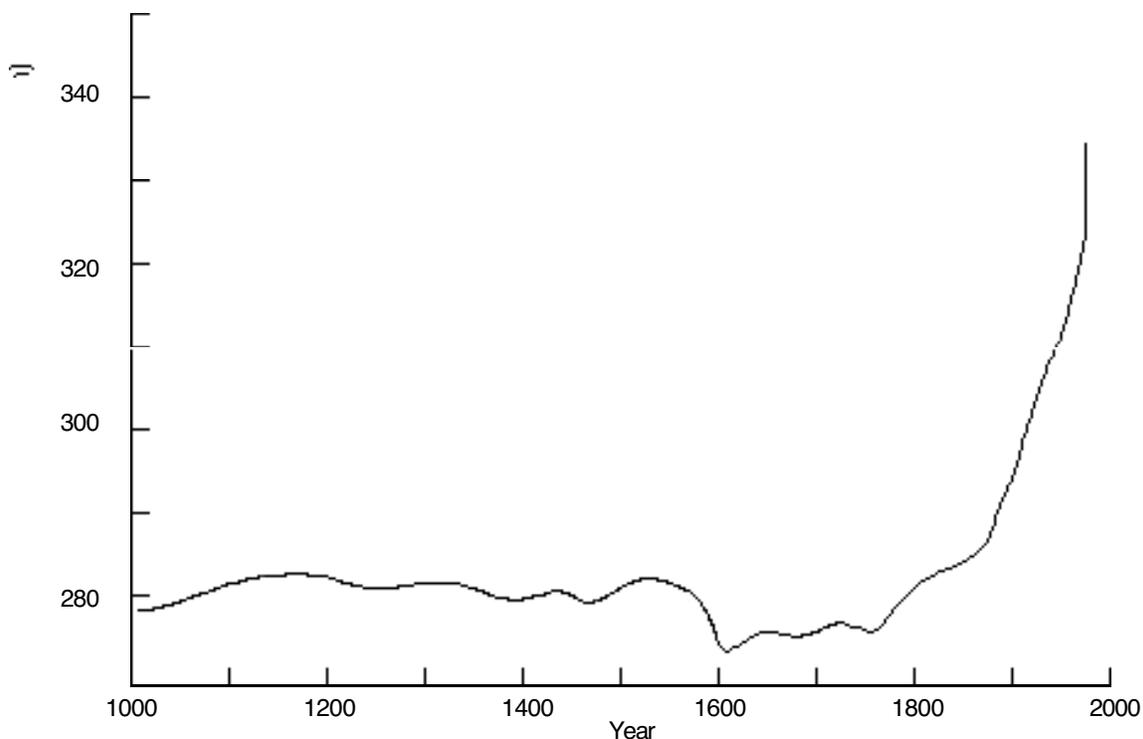


Figure 4: Graph showing concentration of carbon dioxide in atmosphere, Year 1000 CE - Year 2000 CE.

Major sources of anthropogenic carbon dioxide are shown in Figure 5¹³. The approximate contributions of these sources to total CO₂ emissions are:

¹⁰ Image source: <http://www.epa.gov/>

¹¹ Source: Environment Protection Agency. *Inventory of U.S. Greenhouse Gas Emissions and Sinks: 1990-1999*. Washington, DC: National Center on Environmental Publications and Information.

¹² The data that was used to construct Figure 4 was provided by the National Atmospheric and Oceanic Administration, National Climate Data Center, Federal Building, 151 Patton Avenue, Asheville, NC 28801-5001.

¹³ Image sources: (a) Photo credit: David Hoffman. <http://www.hoffmanphotos.demon.co.uk>

- Major transportation activities – 12%.
- Private and commercial vehicles – 22%.
- Factories and home heating – 33%.
- Power plants (especially coal-fired power plants)¹⁴

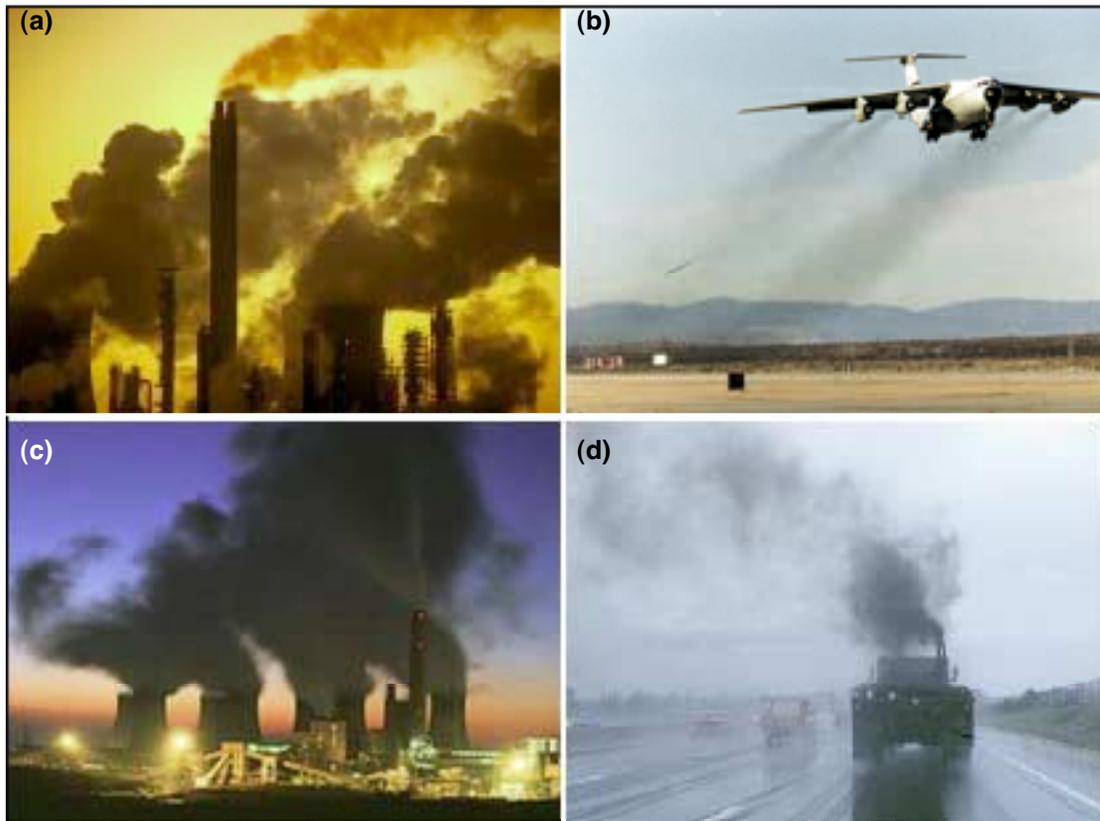


Figure 5: Major sources of anthropogenic carbon dioxide emissions. (a) Factories, (b) mass transport systems, (c) power plants and (d) private and commercial vehicles.

The amount of anthropogenic carbon dioxide released into the atmosphere every year is in the billions of metric tons. Figure 6¹⁵ shows the amounts of anthropogenic carbon dioxide produced by the ten largest emitters during 1996. The United States topped the list with emissions in excess of 5.5 billion tons of carbon dioxide for 1996 alone.

(b) <http://www.dfrc.nasa.gov/> (c) <http://www.physics.nuigalway.ie/images/coal.jpg/>

(d) <http://www.cviog.uga.edu/>

¹⁴ As you may have read while completing Homework Assignment 15, a 1700 MW coal-fired power station will burn about 15.3 million kg of coal on a daily basis. This produces about 51,000 metric tons of the greenhouse gas carbon dioxide (CO₂) and about 140 metric tons of the greenhouse gas nitrous oxide (NO₂) on a *daily* basis.

¹⁵ The data used to compile Figure 6 was obtained from the Environmental Protection Agency, Climate Protection Partnerships Division, US EPA Headquarters, Ariel Rios Building, 1200 Pennsylvania Ave., N.W., Washington, DC 20460.

Emissions of CO₂ (1996)

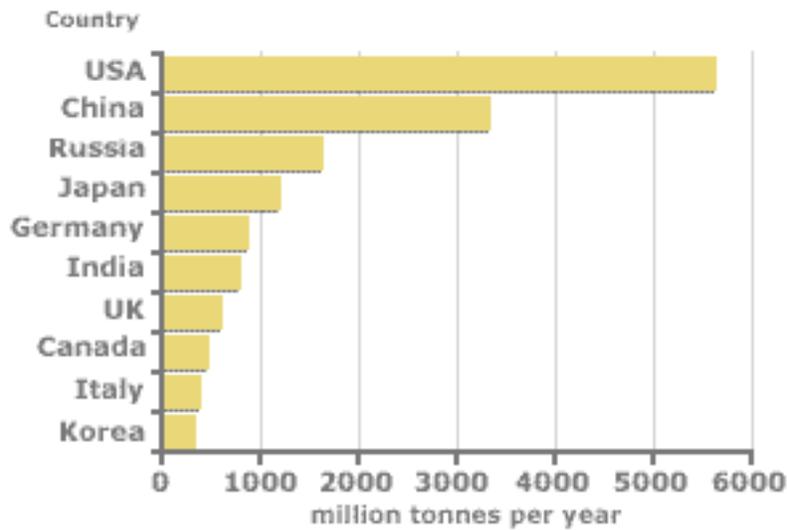


Figure 6: Total emissions of anthropogenic carbon dioxide during 1996 for the ten largest producers of anthropogenic carbon dioxide.



Figure 7: The troposphere and stratosphere as seen from Earth orbit. The Earth's atmosphere is the narrow blue band between the surface of the Earth (lower portion of figure) and space (the black area at the top of the figure).

In this ICE you will use calculus, geometry and integrals to calculate the amount (in units of metric tons) of carbon dioxide in the lower portion of the atmosphere.

Unlike the second most important greenhouse gas (methane, CH₄) that you investigated in an earlier ICE, carbon dioxide spends a long time in the atmosphere. On average, methane spends about 12 years in the atmosphere

before it is removed by some kind of natural process. Carbon dioxide¹⁶, by comparison, spends 150-200 years in the atmosphere before it is removed¹⁷.

¹⁶ Source: Carbon Dioxide Information Analysis Center, Oak Ridge National Laboratory, P.O. Box 2008, Oak Ridge, TN 37831.

¹⁷ The main processes that remove carbon dioxide from the atmosphere are photosynthesis and atmospheric CO₂ dissolving in the waters of the world's oceans. Once in the oceans, some of the CO₂ reacts with

Efforts to reduce anthropogenic methane emissions will likely have a noticeable short-term effect on the concentration of methane in the atmosphere as methane is relatively short-lived as a component of the atmosphere. However, as carbon dioxide spends so long in the atmosphere, we will need to not only reduce current emissions but also implement some means of removing carbon dioxide from the atmosphere if we are to reduce the concentration of CO₂ in the atmosphere. An important starting point in this effort is to ascertain how big of a job this will be, and that is what you will calculate in this ICE.

Figure 7¹⁸ shows the parts of the Earth's atmosphere that is closest to the surface of the earth. These are the troposphere (which extends from the surface of the Earth to an altitude of about 10 km) and the stratosphere (which extends from an altitude of 10 km to an altitude of 40 km). The top of the troposphere is approximately the same as the height of Mount Everest. Almost all human activity (including flights in commercial aircraft) occurs within the troposphere.

The concentration of carbon dioxide changes with altitude. As CO₂ (molar weight 44g) is heavier than air (average molar weight 29g), carbon dioxide tends to be more concentrated near the surface of the Earth and less concentrated higher in the atmosphere. The vast majority (more¹⁹ than 99%) of atmospheric CO₂ is located in the troposphere. Figure 8²⁰ shows how the concentration of CO₂ changes with altitude.

Let x represent the height above the surface of the Earth (in units of kilometers, km) and let $a(x)$ represent the concentration of carbon dioxide in the atmosphere (in units of metric tons per cubic kilometer). Some values of the function $a(x)$ (taken from Figure 8) are given in Table 1 below.

| Height above surface Of Earth (km) | CO ₂ concentration (metric tons/(km) ³) | Height above surface Of Earth (km) | CO ₂ concentration (metric tons/(km) ³) |
|---------------------------------------|---|---------------------------------------|---|
| 2.06 | 67685.56 | 4.41 | 67097.86 |
| 2.68 | 67179.69 | 4.76 | 67056.95 |
| 3.56 | 67161.09 | 5.65 | 67103.44 |
| 3.85 | 67179.69 | 5.94 | 67138.78 |

Table 1: Atmospheric CO₂ concentrations.

elements in the seawater to form insoluble compounds that precipitate. These precipitates sink to the ocean floor where they may become incorporated into the Earth's crust.

¹⁸ Image source: <http://www.nasa.gov/>

¹⁹ Source: <http://imnh.isu.edu/digitalatlas/basics/climate/chem/text/text.htm>

²⁰ The data used to construct Figure 8 were obtained from the National Oceanic and Atmospheric Administration, Cooperative Institute for Research in Environmental Sciences, University of Colorado at Boulder, 216 UCB, Boulder, CO 80309-0216. Strictly speaking, Figure 8 shows the CO₂ concentration over Carr, CO, on May 19, 1994. We are going to make the simplifying assumption that the CO₂ concentration over all of the Earth is similar.

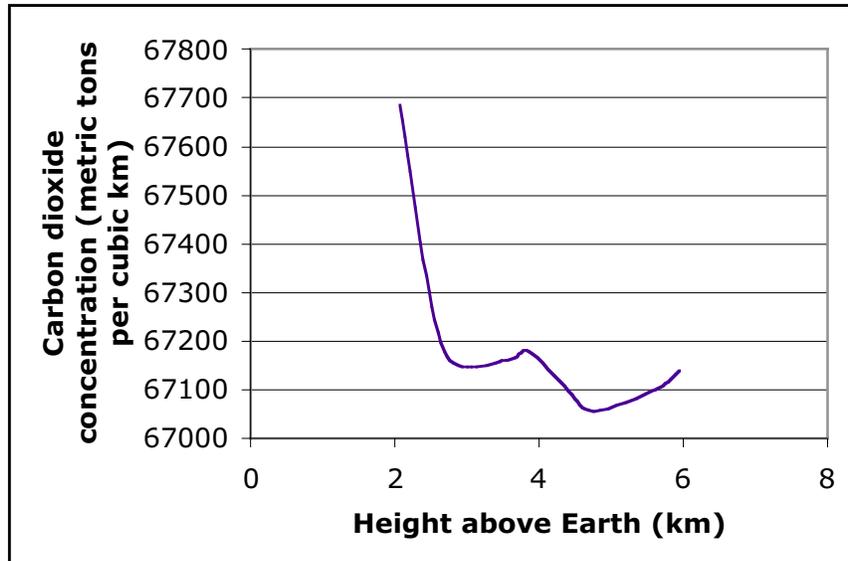


Figure 8: Concentration of atmospheric carbon dioxide versus altitude.

- ***Based on the appearance of the graph in Figure 8, what sort of function would do a reasonable job of giving carbon dioxide concentration as a function of altitude?***

- ***Enter the data from Table 1 into your calculator and use the regression capabilities of your calculator to find an equation for the concentration of carbon dioxide, $a(x)$.***

- **Figure 9 shows a cross-sectional view of the earth surrounded by a thin spherical shell of the troposphere. Find an equation (expressed in terms of x , dx and constants) for the volume of this thin spherical shell.**

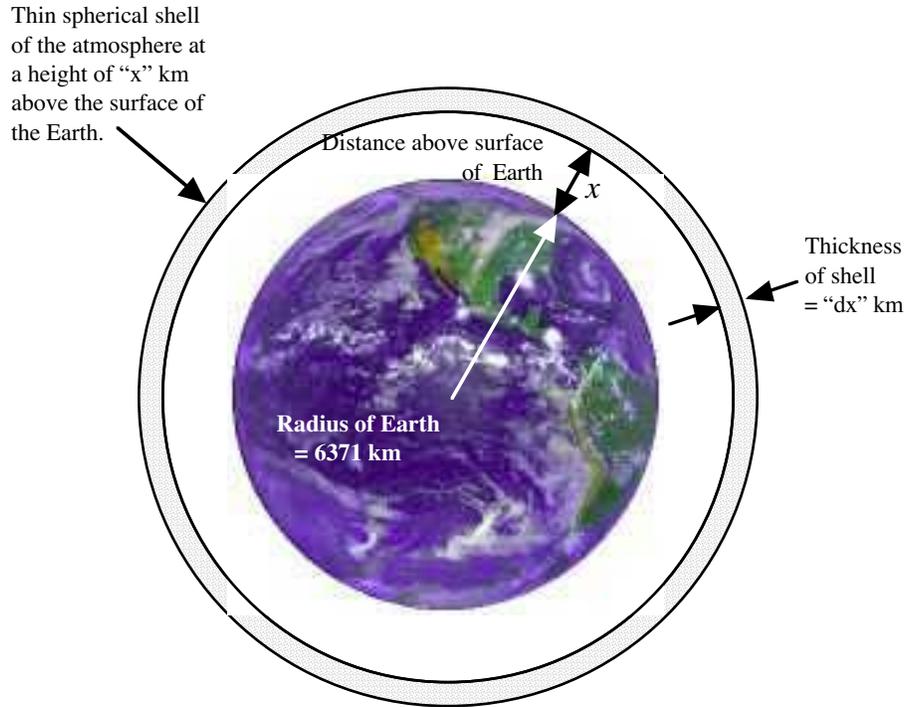


Figure 9: Find an expression for the volume of the thin spherical "shell" of atmosphere that is located at a height of "x" km above the surface of the Earth, and has a thickness of "dx" kilometers.

- **Combine your volume formula with the function $a(x)$ to create an expression that will give the amount of carbon dioxide in the thin spherical shell shown in Figure 9. What are the units of this amount?**

• **The troposphere extends from the surface of the Earth ($x = 0$) to an altitude of 10 km ($x = 10$). Create an integral that will give the total amount (in units of metric tons) of carbon dioxide that is currently in the atmosphere.**

• **Using antiderivatives, find the numerical value of your integral. The number that you get will be the total amount of carbon dioxide currently in the atmosphere (in units of metric tons).**

Epilogue



Figure 10: Forests are a beautiful and profitable way of removing anthropogenic carbon dioxide from the atmosphere.

As noted previously, because CO_2 spends a long time in the atmosphere, simply reducing carbon dioxide emissions will not be sufficient to reverse any effects on climate that anthropogenic CO_2 may cause. Therefore, we will also need to implement some kind of process for removing at least some of the CO_2 that is currently in the atmosphere.

Luckily, nature has already provided an ideal technology for accomplishing this – photosynthesis. Therefore, efforts to reduce anthropogenic CO_2 could be as simple as planting more trees. This is not only easy and relatively cheap to

do, but may also improve the quality of human life by beautifying our surroundings. If the trees planted are commercially valuable species, then efforts to clean up atmospheric CO_2 will not only be cheap, they might actually make large profits through the sale of timber and other wood products.