ICE - The Ratio Test

A very useful test for deciding whether a complicated series converges or not is the ratio test. In order to carry out the ratio test for a series,

$$\sum_{j=1}^{\infty} a_j$$

where the terms a_j are always positive, you could:

1. *Identify the j^{th} term in the series, a_j, and the $(j+1)^{th}$ term, a_{j+1}.*
2. *Form the quotient a_{j+1} / a_j and simplify as much as you can.*
3. *Take the limit of the quotient as $j \to \infty$.*
4. *Interpret the limit:*

The limit is...	The series $\sum_{j=1}^{\infty} a_j$ ...
$0 \le$ limit < 1	converges
limit = 1	no information
limit > 1	diverges

- ***Use the ratio test to decide whether or not the following series converge.***

Series	a_j	a_{j+1} / a_j	Limit of Quotient	Conclusion
$\sum_{k=0}^{\infty} \frac{k+1}{4^k}$				
$\sum_{k=0}^{\infty} \frac{3^k}{k^2+1}$				
$\sum_{k=1}^{\infty} \frac{1}{k}$				
$\sum_{k=0}^{\infty} \frac{2^k}{k!}$				
$\sum_{k=0}^{\infty} \frac{k^2}{2^k}$				

Answers: **(a)** Limit = 1/4. Series converges. **(b)** Limit = 3. Series diverges. **(c)** Limit = 1. Ratio test is inconclusive. (This series actually diverges, although this fact is quite hard to establish.) **(d)** Limit = 0. Series converges. **(e)** Limit = 1/2. Series converges.