



ICE - Sine and Cosine Derivatives

Human beings can survive for seven to nine weeks without any food¹, for less than seven days without water², but for only a few minutes without oxygen. In this ICE³, you will investigate the design of the human respiratory⁴ system to decide how efficient the structure actually is.

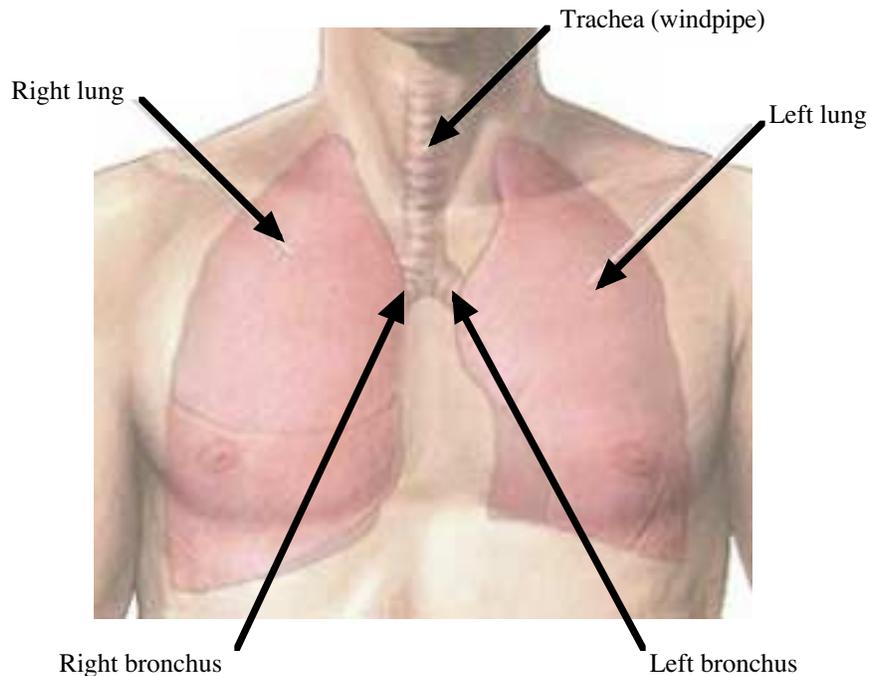


Figure 1: Basic anatomy of the human respiratory system.

When you breathe, air is drawn down the trachea. The trachea splits into two tubes called the left bronchus and the right bronchus. As the CT scan (see Figure 3⁵) shows, the left bronchus is a narrow off-shoot of the right bronchus.

The flow of air through a pipe is modeled by a mathematical equation known as Poiseuille's Law.

¹ Source: Columbia University Health Education Program, <http://www.goaskalice.columbia.edu/>

² Source: World Bank Group, <http://www.worldbank.org/>

³ The version of Poiseuille's Law used in this ICE is adapted from one used to model blood flow in: David Smith and Lawrence Moore. "Calculus. Modeling and Application." D. C. Heath & Co., Lexington, MA, 1996.

⁴ Image source: <http://www.webmd.lycos.com/>

⁵ Image source: Department of Radiology, University of Iowa College of Medicine.

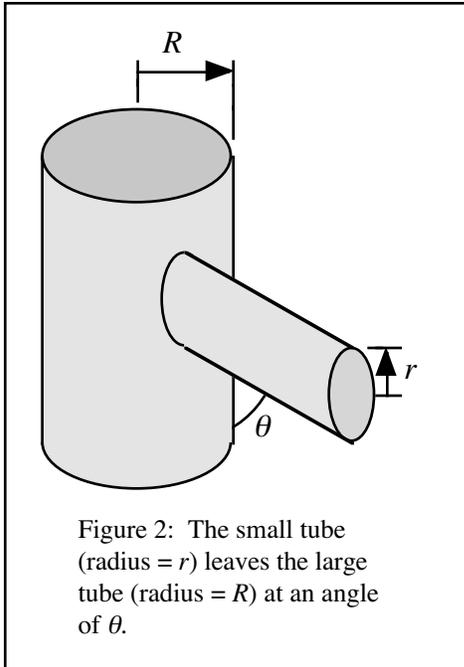


Figure 2: The small tube (radius = r) leaves the large tube (radius = R) at an angle of θ .

When a pipe branches off the air flow becomes disorganized (or turbulent). This turbulence resists the flow of air. Poiseuille's Law predicts that the total resistance to air flow (T) is given by:

$$T = \frac{-\cos(\theta)}{R^4 \cdot \sin(\theta)} + \frac{1}{r^4 \cdot \sin(\theta)}$$

- Measure the values of r and R using the image of the left and right bronchi shown in Figure 3.

- When the radii of the pipes (r and R) have been specified, the variable in the turbulence equation is θ . Find the

derivative of the total resistance to air flow, T .

- Find the value of $\cos(\theta)$ that minimizes the total resistance to air flow (T) into and out of human lungs.

Figure 3⁶ shows a CT scan of the trachea and bronchi of a patient at the University of Iowa College of Medicine⁷.

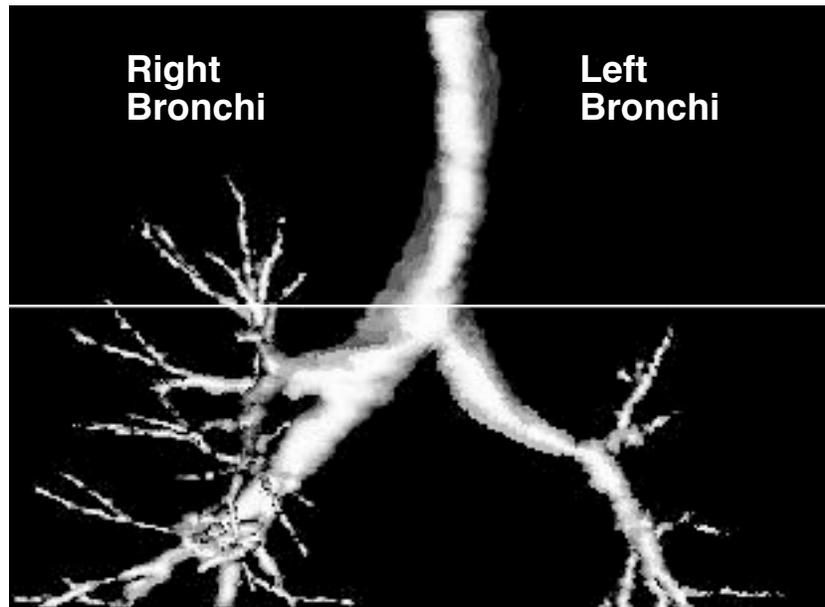


Figure 3: CT scan of a patient showing trachea, left and right major bronchi and secondary bronchi.

- ***Use a protractor to measure the angle between the left and right bronchi in Figure 3.***

- ***Use the inverse cosine function to find the value of θ that minimizes total resistance to air flow. Can Poiseulle's Law explain why the human bronchi branch the way that they do?***

- ***In the ICE, you were asked to find a value that minimized the total resistance to air flow. How did you determine that the value you found was, in fact, a minimum for resistance to air flow?***

⁶ "CT" stands for "Computer Tomography." This is a technique that uses an x-ray scanner and a computer to create a three dimensional picture of structures inside the human body.

⁷ This CT scan was made on February 16, 2001. Image source: The Division of Physiological Imaging, Department of Radiology, University of Iowa College of Medicine.