



## ICE - Sine and Cosine Integrals

**The giant bluefin tuna (*Thunnus thynnus*) is the largest bony fish known to science. This fish can grow to a length of eleven feet and weigh up to 1500 pounds<sup>1</sup>. The bluefin tuna is a remarkably strong fish, and is able to retract its fins and eyes to make it more streamlined. Bluefin tuna have been observed to swim at speeds of up to 55 miles per hour<sup>2</sup>.**

**Bluefin tuna are valued as a food source, especially as sushi and sashimi.**



**A large tuna in excellent condition may bring as much as \$80,000 when sold at auction in Tokyo<sup>3</sup>. Bluefin have been commercially fished in the western Atlantic since the 1960's, with the industry firmly established by the 1980's<sup>4</sup>.**

**Bluefin fishing has become such a lucrative business that commercial tuna fishermen routinely use spotter aircraft to find the fish<sup>5</sup>.**



**Contrary to many perceptions<sup>6</sup>, most tuna fisheries, although heavily exploited are not seriously overfished<sup>7</sup>. This is not the case for bluefin tuna which have been heavily overfished for at least two decades<sup>8</sup>. Based on studies conducted by the International Convention for the Conservation of Atlantic Tuna (ICCAT) and the National Research Council (NRC), the breeding population of Atlantic bluefin tuna fell from approximately 235,000 in 1975 to less than 40,000 in the late 1990's.**

**In 1998, ICCAT proposed an historic plan to limit catch sizes of bluefin tuna to allow the population to**

<sup>1</sup> Source: New England Aquarium ([www.neaq.org](http://www.neaq.org)). The largest bluefin tuna on record was caught by Ken Fraser in Canada during 1979. The fish that Fraser caught weighed 677 kg (1497 pounds).

<sup>2</sup> Source: World Wildlife Fund. ([www.panda.org](http://www.panda.org)).

<sup>3</sup> Source: NASA and the Smithsonian Institution Ocean Planet Project ([seawifs.gsfc.nasa.gov](http://seawifs.gsfc.nasa.gov)).

<sup>4</sup> Source: National Academy of Sciences, National Research Council. *An Assessment of Atlantic Bluefin Tuna*. Washington, DC: National Academy Press, 1994.

<sup>5</sup> Source: World Wildlife Fund.

<sup>6</sup> For example, see: Cole, J. N. "The Vanishing Tuna." *Atlantic Monthly*, Volume 239, p. 50. (Dec. 1976)

<sup>7</sup> Source: Environmental Protection Agency, Revised Final Environmental Impact Statement to accompany Fisheries Management Plan for Highly Migratory Species, 1999.

<sup>8</sup> Source: Buck, Eugene. "Atlantic Bluefin Tuna: International Management of a Shared Resource." The National Council for Science and the Environment, Washington DC, 1995.

recover. In this ICE, you investigate the effects of the ICCAT plan on the population of Atlantic bluefin tuna.

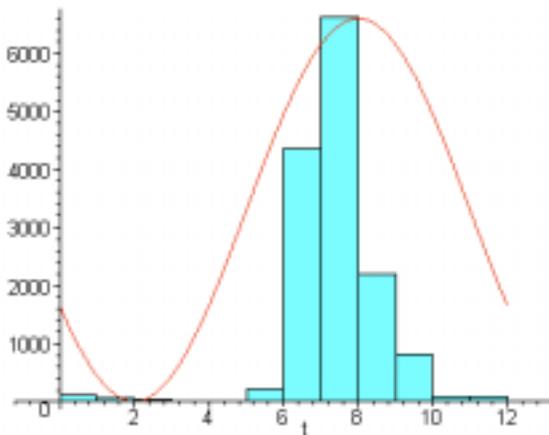


Figure 1: Number of bluefin tuna caught by month, 1998.

The commercial bluefin fishing season runs from June 1 to May 31, or until the quota has been reached<sup>9</sup>. As shown in the histogram (Figure 1, left) most fish are caught between July and October, with relatively few fish harvested during the rest of the year<sup>10</sup>. Regulations governing the size of tuna that can be caught<sup>11</sup> mean that it is mainly sexually mature adults that are harvested.

The curve in Figure 1 is a rough model for the rate at which tuna caught (in tuna caught per month)<sup>12</sup>. The equation for the curve is:

$$\text{Rate} = 3300 - 3300 \cos\left(\frac{\pi}{6}(t - 2)\right).$$

Research programs are now underway to study the bluefin tuna, and in particular whether or not the plan proposed by ICCAT in 1991 will enable the bluefin tuna population to recover or not. Some of these studies involve attaching electronic tags to tuna, which are monitored by satellites. The information collected is relayed to scientists in the US and Canada<sup>13</sup>. Some of the data collected from these studies is shown in Figure 2. The rates of change given in Figure 2 are for sexually mature bluefin tuna and do not include juveniles.

- Describe the trend shown in Figure 2 in words. Use Figure 1 to explain the features of Figure 2.

<sup>9</sup> Source: U.S. Department of Commerce, National Oceanic and Atmospheric Administration, National Marine Fisheries Service. "Small Entity Compliance Guide for the Consolidated Regulations of Atlantic Tuna, Swordfish, Sharks and Billfish." 1999.

<sup>10</sup> Source of data: Personal communication from the National Marine Fisheries Service, Fisheries Statistics and Economics Division, Silver Spring, MD.

<sup>11</sup> Source: U.S. Department of Commerce, National Oceanic and Atmospheric Administration.

<sup>12</sup> Source of data: National Marine Fisheries Service, Fisheries Statistics and Economics Division, and Roberta Holland. "Bluefin tuna prices hit rock bottom." *Boston Business Journal* (September 11, 1998).

<sup>13</sup> Some of this research is being carried out locally by Dr. Molly Lutcavage a marine biologist at the New England Aquarium.

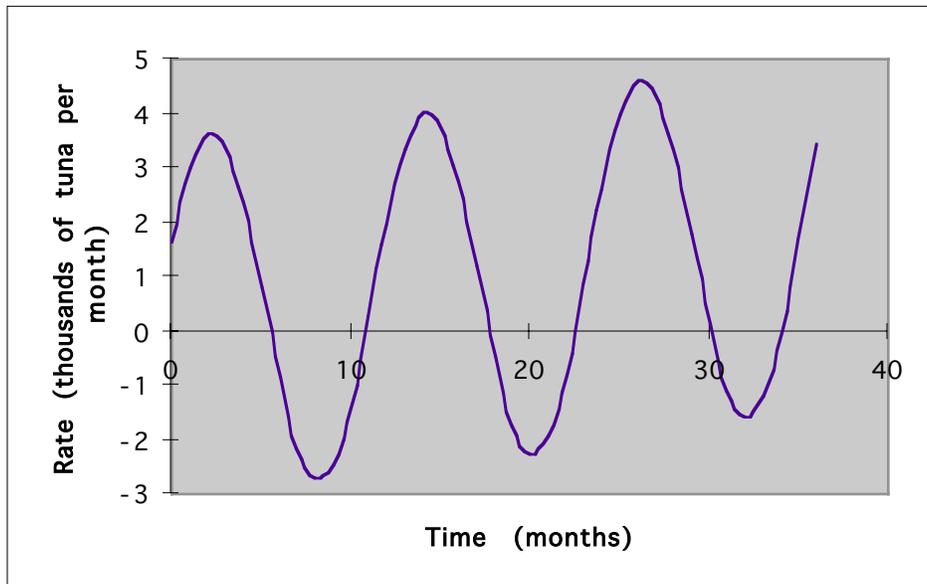


Figure 2: Rate of change of bluefin tuna breeding population (thousands of tuna per month) 1998-2001. (Source: International Commission for Conservation of Atlantic Tuna).

- **Use the symbols  $R(t)$  to denote the rate of change of the breeding population at time  $t$ . (The units of  $R(t)$  are thousands of tuna per month.) When left alone, the tuna population grows at a rate of  $3.5 \cdot e^{0.0875 \cdot t}$  in units of thousands of tuna per month. Find an equation for  $R(t)$ .**

- **In terms of tuna, what does:**

$$\int_0^{10} R(t) dt$$

**represent? What is the numerical value of this quantity?**

