One of the oldest known stone structures in the world is the Step Pyramid of Netjerykhet Djoser[1] located near Saqqara in Egypt (see Figure 1[2]). The Step Pyramid began as a simple bunker-like tomb (a mastaba). A small, four step pyramid was built over the original mastaba, and the final six step pyramid built over this (see Figure 2[3]). The final six step pyramid is 60 meters high.

Figure 1: The Step Pyramid of Netjerykhet Djoser (Zoser) located near Saqqara in Egypt.
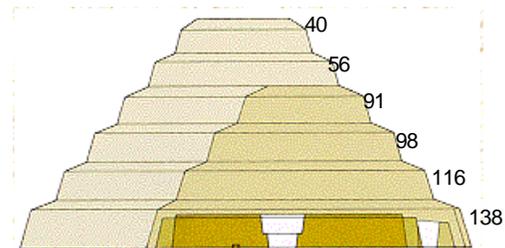

Figure 2: Stages of construction of the Step Pyramid. (The numbers are the average width of each step of the Six Step Pyramid in meters.)

- **In total, about how much stone was used to construct the Step Pyramid?**

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[1] "Djoser" is also referred to as "Zoser" by some scholars.

[2] Image source: www.bikeabout.org

[3] Image source: www.egyptvoyager.com

The Step Pyramid is thought to be the first large pyramid built in Egypt. Later pyramids (such as the Great Pyramid of Khufu[4] - see Figure 3[5] below) had smooth triangular sides.

Figure 3: The Great Pyramid of Khufu (Cheops) located near Giza in Egypt.
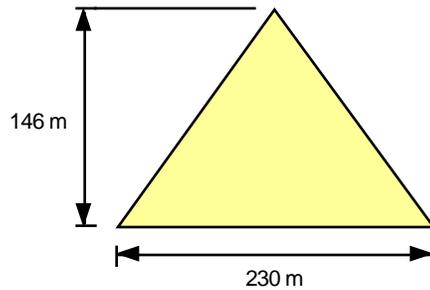

Figure 4: Side view of the Great Pyramid of Khufu (Cheops).

In this part of the ICE, you're going to use geometry and series to approximate the volume of the Great Pyramid of Khufu.

• Use the principle of similar triangles and the diagram in Figure 5 to find a formula for the width, w, of the Great Pyramid at a height of d meters off the ground.
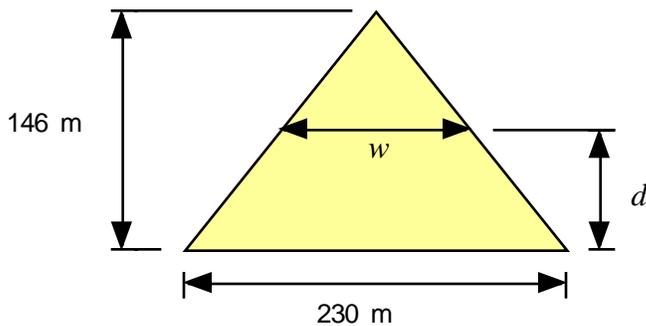

Figure 5: Working out the width (w) of the Great Pyramid at a height (d).

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[4] Also known as the Great Pyramid of Cheops.

[5] Image source: www.bikeabout.org

- **We're going to approximate the Great Pyramid by a step pyramid with the same base (230 m) and the same overall height (146 m), as shown in Figure 6. Use your formula to calculate the volume of each step, and then the total volume.**

Figure 6: The Great Pyramid and an approximation using four steps.

- **Use the following table to work out the (approximate) volume of the great pyramid using a four step pyramid as an approximation.**

Step	Thickness (meters)	Width (meters)	Volume of step (cubic meters)

- **Use the following table to work out the (approximate) volume of the great pyramid using a ten step pyramid as an approximation.**

Step	Thickness (meters)	Width (meters)	Volume of step (cubic meters)

- **Now suppose that the number of steps that you want to use in the approximation is "N" (a positive integer). Carefully examine the entries that you made in the tables for the four step and ten step approximations, and formulate an algebraic expression that will give the volume of an "N" step approximation to the Great Pyramid of Khufu.**

- **Now express the volume of the "N" step approximation using the "Σ" notation for a series.**

- **The formula for the volume of a pyramid such as this is one third of the height time the area of the base. Approximately how much stone was used in the Great Pyramid of Khufu?**