In Class Exercises (ICE) - 2/28/01

Karl Gauss is said to be one of the greatest mathematicians who ever lived. One piece of evidence for this claim is a story about Gauss' school days in Germany.

Figure 1: Portrait of Karl Friedrich Gauss (1777-1855).
(Image source: University of Indiana.)

One morning during math period, Gauss' school master decided that he was sick of teaching his students, so instead of performing at the chalkboard for an hour, he gave the class a problem to work on. The school master told the class to find the sum of the integers from 1 to 100 (inclusive). Thinking that this sum would take the students the entire hour, the school master relaxed with a book.

However, after only a couple of minutes, young Gauss raised his hand.

"Yes, Gauss, what is it?" barked the school master in German, not even trying to conceal his annoyance.

"Is the answer 5050?" asked young Gauss in respectful German.

The schoolmaster hurumphed, slammed his book down, and checked the answers in the back of his "Teachers' Edition." To his amazement, the answer *was* 5050.

- ***How did Gauss figure out this sum so quickly?***

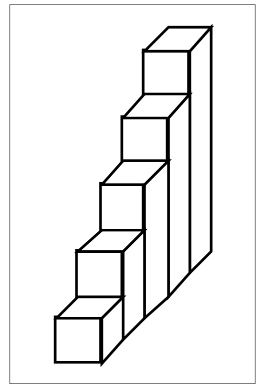
In mathematical notation, the sum of all the whole numbers between 1 and 'n' can be written as:

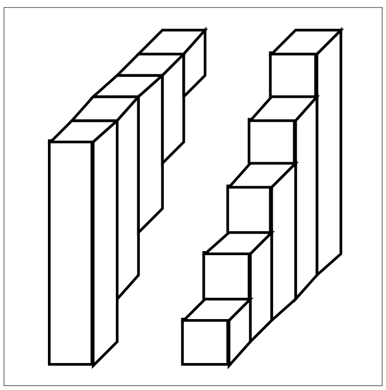
$$\sum_{j=1}^{n} j$$

- ***Can you generalize Gauss' reasoning to find a formula for this sum?***

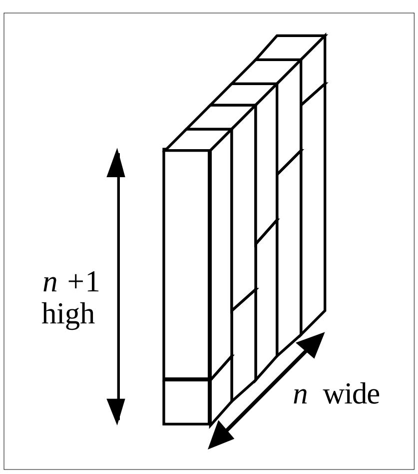
We have seen one way of establishing a formula for the sum of a series - by finding a cunning way to look at the terms in the series. Another way of finding such a formula is to represent the mathematical proposition in multiple ways (say algebra and pictures).

$$\sum_{j=1}^{n} j$$





$$\left(\sum_{j=1}^{n} j\right) + \left(\sum_{j=1}^{n} j\right) = 2 \cdot \sum_{j=1}^{n} j$$



$$2 \cdot \sum_{j=1}^{n} j = n \cdot (n+1)$$