In Class Exercises (ICE) - 2/5/01

According to government statistics[1], revolving consumer debt reached an all-time high in 2000 of 558.6 billion dollars. This debt consists mainly of unpaid balances on credit card accounts. Credit card debt usually starts out with an initial debt (D_0) that the customer cannot pay off[2], which then accumulates because the credit card company charges a high rate of interest on the outstanding debt. If the customer does not make any payments, then the debt is given as a function of time:

$$D(t) = D_0 \cdot \left(1 + \frac{r}{12}\right)^{12t}$$

where $D(t)$ is the outstanding debt after 't' years, D_0 is the initial amount of debt, and 'r' is the annual interest rate (expressed as a decimal).

- ***There is a "12" in the base and in the exponent of this exponential function, $D(t)$. In terms of how credit card companies calculate interest, explain why these numbers appear in the formula.***

- ***Suppose that a customer starts out with a debt of $1000, and the annual interest rate is 15.90%. Find the value of $D'(5)$. How can you interpret the practical significance of this number?***

---

[1] Source: Board of Governors of the Federal Reserve System, Monthly Federal Reserve Bulletin.

[2] See **http://cbsnews.com/now/story/0,1597,265630-412,00.shtml** for a recent story on credit card comapnies and college students.

Although fixed annual percentage interest rates (APR's) are becoming more common, many credit card companies still charge a variable interest rate. One prominent bank computes their APR by taking the "Prime Rate" published at the end of each month in the Wall Street Journal and adding a margin of 5.9%. For example, if the prime rate is 10%, then the credit card company would charge an APR of 15.9%. The accumulation of debt in this kind of situation is better described by a formula of the form:

$$D(t) = D_0 \cdot \left(1 + \frac{r(t)}{12}\right)^{12t}.$$

- ***Several values of the "Prime Rate" are given in Table 1 below[3]. Use the data in Table 1 to find a formula for*** $r(t)$***.***

Date	1/31/98	1/31/99	1/31/00
Wall Street Journal Prime Rate (%)	7.75	8.50	9.50
t	0	1	2
$r(t)$	0.0775	0.0812	0.0858

Table 1: Wall Street Journal Prime Interest Rate (January 1998, 1999, 2000).

- ***Can you find the derivative of this debt function using the same derivative rules as before? Explain why or why not.***

- ***Complete the entries in the table below and use this to approximate the value of*** $D'(5)$***.***

t	$D(t)$	$\frac{D(t) - D(5)}{t - 5}$
4.9		
4.99		
4.999		

- ***Use the technique of logarithmic differentiation to calculate*** $D'(5)$ ***using derivative rules.***

[3] Source: Enterprise Data Access (www.enth.com)