



## In Class Exercises (ICE) - 2/7/01

The goal here is to find formulas for  $\frac{dy}{dx}$ . In some cases, you may be able to re-arrange the given equation to make 'y' the subject, but in other cases this is mathematically impossible. Nevertheless, it is always possible to find an expression for  $\frac{dy}{dx}$ .

- $x^2 \cdot y + e^{x \cdot y} = 1$ .

- $x^4 + 4x^2 \cdot y^2 + 4y^4 = 25$ .

- $\ln(x \cdot y^2) + \ln(x) \cdot y^2 = 100$ .

- $y^2 + 2x \cdot y - 3x^2 = 0$ .

**Solutions:** (a)  $dy/dx = (-2xy - ye^{xy})/(x^2 + xe^{xy})$ . (b)  $dy/dx = (-4x^3 - 8xy^2)/(8x^2y + 16y^3)$   
(c)  $dy/dx = (-1/x - y^2/x)/(2/y + \ln(x) \cdot 2y)$  (d)  $dy/dx = (6x - 2y)/(2y)$

**The equation,**

$$x^2 + 2x \cdot y + 3y^2 = 2$$

**defines the elliptical curve pictured below in Figure 1 below.**

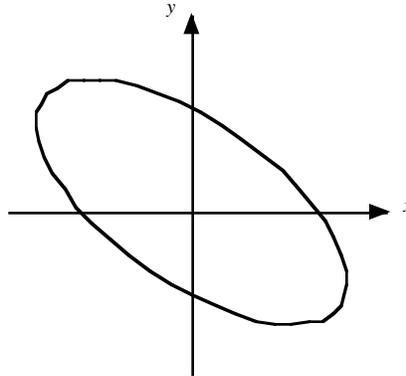


Figure 1: Ellipse defined by the equation  $x^2 + 2xy + 3y^2 = 2$ .

- **Find a formula for  $\frac{dy}{dx}$ .**
- **Verify that the point  $(x, y) = (0, \sqrt{\frac{2}{3}})$  lies on the curve, and calculate the equation of the tangent line to this point on the curve.**
- **The picture of the ellipse shows that there are several places where the tangent line is horizontal. Find the coordinates of one of these points.**
- **The picture of the ellipse shows that there are several places where the tangent line is vertical. Find the coordinates of one of these points.**

**Solutions:** (a)  $\frac{dy}{dx} = \frac{-2xy - ye^{xy}}{x^2 + xe^{xy}}$ . (b)  $\frac{dy}{dx} = \frac{-4x^3 - 8xy^2}{(8x^2y + 16y^3)}$   
(c)  $\frac{dy}{dx} = \frac{-1/x - y^2/x}{(2/y + \ln(x) \cdot 2y)}$  (d)  $\frac{dy}{dx} = \frac{(6x - 2y)}{(2y)}$