Problem 4

a) Let t be the time (in months) and $A(t)$ the amount (in grams) of radioactive waste remaining. Then:

$$A(t) = 5\cdot\left(\frac{1}{2}\right)^{\frac{t}{8}}.$$

b) The completed table is shown below.

Dump number	Total amount of radioactive material in the site after the dump (g)
1	5
2	$5\cdot\left(\tfrac{1}{2}\right)^{\frac{1}{8}}+5$
3	$5\cdot\left(\tfrac{1}{2}\right)^{\frac{2}{8}}+5\cdot\left(\tfrac{1}{2}\right)^{\frac{1}{8}}+5$
4	$5\cdot\left(\tfrac{1}{2}\right)^{\frac{3}{8}}+5\cdot\left(\tfrac{1}{2}\right)^{\frac{2}{8}}+5\cdot\left(\tfrac{1}{2}\right)^{\frac{1}{8}}+5$

c) After the 30th dump, the amount of radioactive waste in the dump site (in grams) is:

$$5\cdot\left(\tfrac{1}{2}\right)^{\frac{29}{8}}+5\cdot\left(\tfrac{1}{2}\right)^{\frac{28}{8}}+\ldots+5\cdot\left(\tfrac{1}{2}\right)^{\frac{3}{8}}+5\cdot\left(\tfrac{1}{2}\right)^{\frac{2}{8}}+5\cdot\left(\tfrac{1}{2}\right)^{\frac{1}{8}}+5.$$

Putting this into closed form and evaluating gives: $\dfrac{5-5\cdot\left(\tfrac{1}{2}\right)^{\frac{30}{8}}}{1-\left(\tfrac{1}{2}\right)^{\frac{1}{8}}}\approx 55.76g$ of radioactive waste.

d) After a total of n dumps, the amount of radioactive waste in the dump site will be given by the series:

$$5\cdot\left(\tfrac{1}{2}\right)^{\frac{n-1}{8}}+5\cdot\left(\tfrac{1}{2}\right)^{\frac{n-2}{8}}+\ldots+5\cdot\left(\tfrac{1}{2}\right)^{\frac{3}{8}}+5\cdot\left(\tfrac{1}{2}\right)^{\frac{2}{8}}+5\cdot\left(\tfrac{1}{2}\right)^{\frac{1}{8}}+5.$$

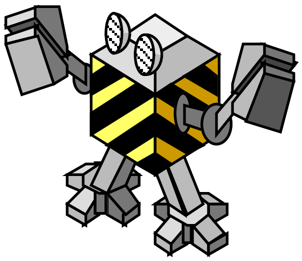
Putting this into closed form gives:

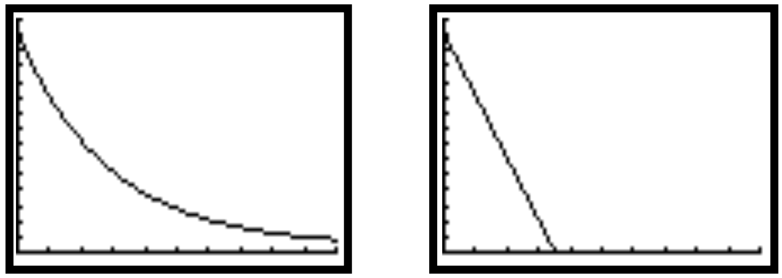
$$\frac{5-5\cdot\left(\tfrac{1}{2}\right)^{\frac{n}{8}}}{1-\left(\tfrac{1}{2}\right)^{\frac{1}{8}}}.$$

One way to solve this problem is to set this closed form equal to 60g and then see if it is possible to find a value of n that satisfies the equation. If it is possible to find a value of n that satisfies the equation, then the dump site will eventually contain 60g of radioactive waste and the company will have violated the ordinances of Columbus, Ohio. If you carry this out, then you find that n = 63.59 works in the equation.

Problem 5

a) The two graphs are shown below. The points are not marked on the graphs due to limitations of word-processing technology, but the two easiest points to label would be (0, 140000) and (1, 100000).





Graphs of robot value versus time for depreciation methods 1 and 2. In each case, the viewing window is [0, 10] by [0, 150000].

b) Let t be the time (in years) since the robot was purchased. Let $V(t)$ be the value of the robot (in dollars) at time t.

$$\frac{dV}{dt} = k(V - 4000)$$

where k is a constant.

c) To solve the above differential equation, make the substitution: $P = V - 4000$.

$$\frac{dP}{dt} = kP$$

$$P = P_0 \cdot e^{kt} \qquad \text{where } P_0 \text{ is a constant.}$$

$$V - 4000 = P_0 \cdot e^{kt}$$

$$V = 4000 + P_0 \cdot e^{kt}.$$

To determine the value of P_0 substitute $t = 0$ and $V = 140000$. This gives $P_0 = 136000$. To determine the value of k, substitute $t = 1$ and $V = 100000$. This gives $k = -0.348$. The final answer is:

$$V = 4000 + 136000 \cdot e^{-0.348t}.$$

d) The formula given for V will eventually approach 4000 as t gets really, really big. This is consistent with the graph of V versus t, which shows a horizontal asymptote of height 4000.

e) You are looking for a linear function. $V = -40000t + 140000$.

f) The differential equation is:

$$\frac{dV}{dt} = -40000.$$

g) The idea here is to substitute the formula from part (e) into the differential equation from part (f) and verify that everything works out. Substituting the formula from (e) in the left hand side of the differential equation from (f) gives:

$$\frac{dV}{dt} = \frac{d}{dt}(-40000t + 140000) = -40000.$$

This agrees perfectly with the right hand side of the differential equation from (f).