

Homework Assignment 17: Solutions

1. The measurements of the two radii and the angle from Figure 3 are shown in the table below.

Quantity	Value
Radius of right bronchus (R)	4 mm
Radius of left bronchus (r)	2.5 mm
Angle between bronchi	82°

Substituting the measured values of r and R into the formula for air resistance, $T(\theta)$, gives the following:

$$T(\theta) = \frac{-\cos(\theta)}{(4)^4 \cdot \sin(\theta)} + \frac{1}{(2.5)^4 \cdot \sin(\theta)}.$$

2. The main differentiation rule needed for the calculation of the derivative of air resistance, $T'(\theta)$, is the quotient rule. Applying the Quotient rule to the formula from Question 1 gives:

$$T'(\theta) = \frac{(4)^4 \cdot \sin^2(\theta) + (4)^4 \cdot \cos^2(\theta)}{(4)^8 \cdot \sin^2(\theta)} + \frac{-(2.5)^4 \cdot \cos(\theta)}{(2.5)^8 \cdot \sin^2(\theta)}.$$

Simplifying this formula (using the identity $\sin^2(\theta) + \cos^2(\theta) = 1$) gives:

$$T'(\theta) = \frac{1}{(4)^4 \cdot \sin^2(\theta)} + \frac{-\cos(\theta)}{(2.5)^4 \cdot \sin^2(\theta)}.$$

3. To locate the critical points for air resistance you can set the derivative, $T'(\theta)$, from Question 2 equal to zero and solve for θ . This calculation is shown below.

Step 1: Rearrange the equation to make cos the subject.

$$T'(\theta) = \frac{1}{(4)^4 \cdot \sin^2(\theta)} + \frac{-\cos(\theta)}{(2.5)^4 \cdot \sin^2(\theta)} = 0$$

$$\frac{1}{(4)^4 \cdot \sin^2(\theta)} = \frac{\cos(\theta)}{(2.5)^4 \cdot \sin^2(\theta)}$$

$$\cos(\theta) = \frac{(2.5)^4 \cdot \sin^2(\theta)}{(4)^4 \cdot \sin^2(\theta)} = \frac{(2.5)^4}{(4)^4} = \left(\frac{2.5}{4}\right)^4 = (0.625)^4$$

Step 2: Use inverse cos to find one solution of the equation.

$$\cos(\theta) = (0.625)^4$$

$$\cos^{-1}(\cos(\theta)) = \cos^{-1}((0.625)^4)$$

$$\theta = \cos^{-1}((0.625)^4) = 1.41761 \text{ radians}$$

Step 3: Locate all solutions that lie in one repetition of the function.

The other solution that lies in the first repetition of the function $y = \cos(\theta)$ is $\theta = 2\pi - 1.41761$ radians.

Step 4: Locate all solutions by adding integer multiple of the period to the solutions found thus far.

The period of the function $y = \cos(\theta)$ is equal to 2π . Therefore, if k represents an integer, then *all* critical points of the function $T(\theta)$ are located at the points:

$$\theta = 1.41761 + k \cdot 2\pi \text{ radians.}$$

$$\theta = (2\pi - 1.41761) + k \cdot 2\pi \text{ radians.}$$

4. The angle of the very first critical point calculated is $\theta = 1.41761$ radians, which is the same as 81.22 degrees. Therefore, the angle measured in Question 1 is very close to the first critical point.
5. No matter what the structure of the belief system, you should expect the critical point located at $\theta = 1.41761$ radians to be a local **minimum**. Two possible arguments for this are given below.
 - **Divine Creation (Judeo-Christian tradition):** In the account of the creation of humankind offered in Genesis, the LORD says: “Let Us make him in our image, according to our likeness.” (Genesis 1:26.) Although the Genesis account does not dwell on the anatomical details of humankind’s creation, it is not impossible to argue that humankind, made in the likeness of the LORD, would presumably be afforded every anatomical advantage possible in His design. An obvious anatomical advantage is to make it possible for humans to obtain the oxygen that they need to live in as effortless a process as possible. Hence, it

would not be surprising to learn that the human respiratory system was designed to minimize air resistance.

- **Evolutionary biology:** Resistance to air flow in the trachea and bronchi interferes with the exchange of oxygen and waste gases in the respiratory system, and therefore with the process of respiration. Any organism that experienced less resistance to air flow as it breathed could have a survival advantage over its competitors with more labored breathing in that it would not have to expend as much efficiency to breathe, and would have an easier time supplying the body with oxygen when this was most needed – such as when running from predators, securing resources such as land and food, or competing for mates. If easier breathing did have a survival advantage, and the angle at which the bronchi has a genetic basis that can be passed on to offspring, then organisms with a bronchi angle that minimized air resistance should out-compete other organisms. Eventually, most people would be descended from an ancestor with a bronchial angle that was close to optimum for minimizing air resistance and easing breathing as much as possible.

To check the nature of the critical point located at $\theta = 1.41761$ radians, you can evaluate the first derivative, $T'(\theta)$, slightly to the left and slightly to the right of the critical point and examine the pattern of signs.

θ	1.41	1.41761	1.42
$T'(\theta)$	-0.000197	0	+0.000062

As the first derivative is negative to the left of the critical point and positive to the right of the critical point, the critical point is a minimum.