Homework Assignment 5: Solutions

1. If you were able to make a 20% down payment, this means that you were able to come up with 649,000×0.2 = \$129,800. This leaves 649,000−129,800 = \$519,200 to be financed with a mortgage.

If you did not have to pay any interest on the loan, and simply had to spread \$519,200 over 360 (30 years with 12 months in each year) the monthly mortgage payment would be:

$$\textit{Monthly mortgage payment} = \frac{519000}{360} = \$1442.22 \text{ per month.}$$

The figure of \$1442.22 per month is likely to be a considerable *under*-estimate of the actual monthly mortgage repayment. This is because the interest on the mortgage will be considerable, and this interest is added into the actual mortgage repayments, which will make them larger than \$1442.22.

2. The completed version of Table 2 is shown below.

Time (months)	Amount still owed (\$)
0	519200
1	$519200 \cdot (1+\frac{0.06875}{12}) - M$
2	$519200 \cdot (1+\frac{0.06875}{12})^2 - M \cdot (1+\frac{0.06875}{12}) - M$
3	$519200 \cdot (1+\frac{0.06875}{12})^3 - M \cdot (1+\frac{0.06875}{12})^2 - M \cdot (1+\frac{0.06875}{12}) - M$
N	$519200 \cdot (1+\frac{0.06875}{12})^N - M \cdot (1+\frac{0.06875}{12})^{N-1} - \ldots - M \cdot (1+\frac{0.06875}{12}) - M$

Table 2.

3. The idea of a 30 year mortgage is that after 30 years you owe nothing – that is when N = 360 the outstanding balance is zero. Substituting N = 360 into the expression from the last row of Table 2 and setting this equal to zero gives:

$$519200 \cdot (1+\frac{0.06875}{12})^{360} - M \cdot (1+\frac{0.06875}{12})^{359} - \ldots - M \cdot (1+\frac{0.06875}{12}) - M = 0.$$

The ultimate goal of this calculation is to solve for M, the monthly payment. Rearranging the above equation to get ever term involving M on one side of the equation gives:

$$519200 \cdot (1 + \tfrac{0.06875}{12})^{360} = M + M \cdot (1 + \tfrac{0.06875}{12}) + \ldots + M \cdot (1 + \tfrac{0.06875}{12})^{359}.$$

The right hand side of this equation is a finite geometric series with:

- Initial value $a = M$.
- Multiplicative factor $r = (1 + 0.06875/12)$.
- A total of 360 terms added together.

Using the summation formula for a finite geometric series to sum the right hand side of the equation gives:

$$519200 \cdot (1 + \tfrac{0.06875}{12})^{360} = \frac{M \cdot [1 - (1 + \tfrac{0.06875}{12})^{360}]}{1 - (1 + \tfrac{0.06875}{12})}.$$

Rearranging this to make M the subject and evaluating on a calculator gives:

$$M = \$3410.77.$$

Note that this is greater than the estimate of \$1442.22. That fact alone is not really enough to confirm that \$3410.77, but it is encouraging. If you had obtained an answer of \$100.00 (or something else smaller than the estimate of \$1442.22) then you would know that had definitely made a mistake.

4. A five percent down payment (\$32,450.00) leaves \$616,550.00 to be financed with a thirty-year mortgage. The calculation to determine the monthly repayment under these circumstances is exactly the same as the calculation performed in Question (d) except that 616550 is used in the place of 519,000. After setting up the geometric series and using the summation formula, the equation that gives the monthly payment M will be:

$$616550 \cdot (1 + \tfrac{0.06875}{12})^{360} = \frac{M \cdot [1 - (1 + \tfrac{0.06875}{12})^{360}]}{1 - (1 + \tfrac{0.06875}{12})}.$$

Solving for M and evaluating on a calculator gives: $M = \$4050.29$.

To see how the total amount paid to the lender is different in the two situations, consider the following table.

Down payment (%)	Down payment ($)	Total repayments ($) (360 months)	Total paid to lender ($)
20	129800	1227877.20	1357677.20
5	32450	1458104.40	1490554.40

The entries in the table show that in total, making a larger down payment at the beginning of the loan can make a difference to the total amount paid to the lender. In this case, if you were only able to come up with a 5% down payment then you would pay (1490554.40–1357677.20) = $132,877.20 more than if you had paid a 20% down payment. Generally speaking, if you can make a larger down payment you will realize significant savings over the entire course of the loan. Therefore, the best strategy when obtaining a mortgage is to make the largest down payment that you can possibly afford.

5. When calculating the monthly payment, M, for a 15 year mortgage two things change in the calculation from Part (d). First, as there are only 180 months in 15 years the exponents are 180, not 360. Second, 15 year mortgages tend to get better interest rates than 30-year mortgages. Using the data provided in Table 1, we should use 0.06375 (nominal interest of 6.375%) in place of 0.06875. After setting up the geometric series and using the summation formula, the equation giving M in this case will be:

$$519200 \cdot (1 + \tfrac{0.06375}{12})^{180} = \frac{M \cdot [1 - (1 + \tfrac{0.06375}{12})^{180}]}{1 - (1 + \tfrac{0.06375}{12})}.$$

Rearranging to make M the subject of the equation and evaluating on a calculator gives: M = $4487.19. This is considerably higher than the $3410.77 monthly payment on a 30-year mortgage.

(As an aside with some real-world implications, note that mortgage lenders often stipulate that the monthly repayment may not be more than one third of the homebuyer's income. Therefore, you would normally need an annual income of about $161,500.00 in order to be approved for such a mortgage. Even the more modest 30-year mortgage would probably require an annual household income of about $123,000.00 for approval.)

Despite the fact that the monthly payment is higher, the 15-year mortgage (if you can afford it) offers considerable savings over the duration of the mortgage. The following table compares the 15 and 30-year mortgages.

Length of mortgage (years)	Down payment ($)	Total repayments ($) (entire mortgage)	Total paid to lender ($)
30	129800	1227877.20	1357677.20
15	129800	807694.20	937494.20

If you can afford the monthly payments, the 15 year mortgage will save you (1357677.20 − 937494.20) = $420,183.00. Generally speaking, a shorter mortgage will result in higher monthly payments. However, if you can afford the higher monthly payments a shorter mortgage is usually preferable as it will save you a **lot** of money over the duration of the loan.

Extra Credit

In this question, the set-up is exactly the same as in Question 3, the only thing that changes is that you use 0.07 instead of 0.06875 (i.e. a nominal interest rate of 7% instead of a nominal interest rate of 6.875%. After setting up the geometric series and summing, the equation that gives the monthly payment *M* will be:

$$519200 \cdot (1+\tfrac{0.07}{12})^{360} = \frac{M \cdot [1-(1+\tfrac{0.07}{12})^{360}]}{1-(1+\tfrac{0.07}{12})}.$$

Solving for *M* and evaluating on a calculator gives: *M* = $3454.25. This is a slightly higher (3545.25 − 3410.77 = $43.48) monthly payment than the MortgageSelect loan. Over the course of the loan, however, the East/West loan would cost about $15652.99 more than the MortgageSelect loan. Therefore, very small differences in the interest rate can make a considerable difference to the amount that you need to pay to the lender over the entire length of the loan. Clearly, when obtaining a loan you want to take the loan that has the lowest interest rate.