Appendix F - To find limits of indet. form, we can use L'Hôpital's Rul

We use L'Hôpital's rule only when $\lim\limits_{x\to a} \frac{f(x)}{g(x)} = \frac{0}{0}$ or $\frac{\infty}{\infty}$

- otherwise we can't use l'Hôpital's, or must 1st tranform the limit into the form $\frac{0}{0}$ or $\frac{\infty}{\infty}$

L'Hôpital: $\lim\limits_{x\to a} \frac{f(x)}{g(x)} = \lim\limits_{x\to a} \frac{f'(x)}{g'(x)}$

Examples

① ex of $\frac{0}{0}$: $\lim\limits_{x\to 0} \frac{\sin x}{x} = \frac{0}{0}$ · use l'Hôp.

$= \lim\limits_{x\to 0} \frac{\cos x}{1} = \boxed{1}$

② $\frac{\infty}{\infty}$: $\lim\limits_{x\to\infty} \frac{e^x}{x^2} = \frac{\infty}{\infty}$ · use l'Hôp

$= \lim\limits_{x\to\infty} \frac{e^x}{2x} = \frac{\infty}{\infty}$ · use l'Hôp again

$= \lim\limits_{x\to\infty} \frac{e^x}{2} = \boxed{\infty}$

③ $\frac{\infty}{\infty}$ w/ polynomials

a) $\lim\limits_{x\to\infty} \frac{x^2+1}{3x} = \frac{\infty}{\infty}$

$= \lim\limits_{x\to\infty} \frac{2x}{3} = \boxed{\infty}$

· can use l'Hôp

· also recognize that w/ rational f's

- if $\deg.(f(x)) > \deg.(g(x))$ $\lim\limits_{x\to\infty} \frac{f(x)}{g(x)} = \boxed{\infty}$

b) $\lim\limits_{x\to\infty} \frac{3x}{x^2+1} = \frac{\infty}{\infty}$

$= \lim\limits_{x\to\infty} \frac{3}{2x} = \boxed{0}$

- if $\deg(f(x)) < \deg(g(x))$ $\lim\limits_{x\to\infty} \frac{f(x)}{g(x)} = \boxed{0}$

c) $\lim\limits_{x\to\infty} \frac{3x}{x+1} = \frac{\infty}{\infty}$

$= \lim\limits_{x\to\infty} \frac{3}{1} = \boxed{3}$

- if $\deg(f(x)) = \deg(g(x))$ $\lim\limits_{x\to\infty} \frac{f(x)}{g(x)} =$ ratio of leading coefficients

④ $\infty \cdot 0$ $\lim\limits_{x\to 0^+} x \cot x = 0 \cdot \infty$ · indeterminate form, but can't use l'Hôp ye

$= \lim\limits_{x\to 0^+} \frac{x}{\tan x} = \frac{0}{0}$ · now can use l'Hôp

$= \lim\limits_{x\to 0^+} \frac{1}{\sec^2 x} = \frac{1}{1} = \boxed{1}$

⑤ $1^\infty, 0^0, \infty^0, 0^\infty$ - "indeterminate powers" - for these use logarithms 1st.

let L = the lim. $L = \lim\limits_{x\to\infty} x^{\frac{1}{x}} = \infty^0$ · can't use l'Hôp → take ln of "L"

$\ln L = \lim\limits_{x\to\infty} [\ln x^{\frac{1}{x}}] = \lim\limits_{x\to\infty} \frac{\ln x}{x}$ · now use l'Hôp.

(5) cont... $\lim_{x\to\infty}$ ... $\lim_{x\to\infty}$ $\frac{\ }{x}$

$= \lim_{x\to\infty} \frac{\frac{1}{x}}{1} = \boxed{0}$ ← remember this is the ln of the limit

$\ln L = 0$

$\lim_{x\to\infty} x^{\frac{1}{x}} = L = e^0 = \boxed{1}$ ← so ~~the~~ e^0 is the actual limit.

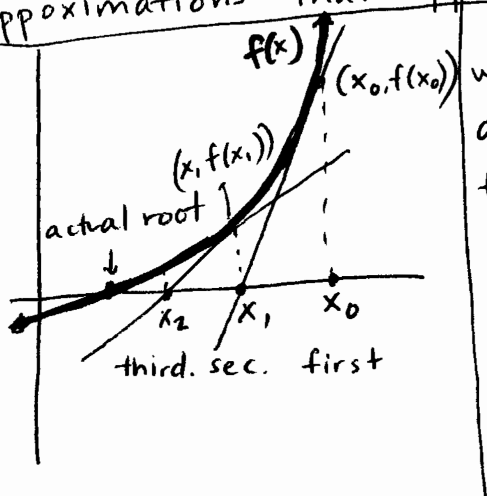
see common mistakes for more examples...

(5) $\underline{1^\infty}$ $\lim_{x\to\infty} \left(1+\frac{a}{x}\right)^{bx} = \boxed{e^{ab}}$

Appendix G

Sometimes exact solutions to the eq. $f(x)=0$ are hard to find. Roots are easy to find when $f(x)=(x-2)(x+1)$ ~~a~~ for example. but when they are hard, we can use Newton's Method.

Newton's method uses tangent lines to give a series of appoximations that approach the sol'n:



we start with a guess, x_0 and using pt.-slope eq. we find the formula for first ~~ea.~~ approximation is

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)} \qquad f'(x_0) \neq 0$$

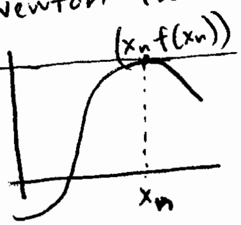
for ea. appoximation after:

$$\boxed{x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}} \qquad f'(x_n) \neq 0$$

Limitations to Newton's Meth.

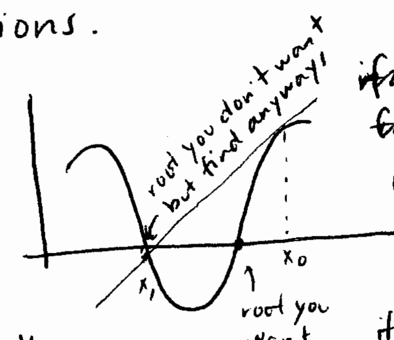
Newton fails ~~if~~ under certain conditions.

(1)



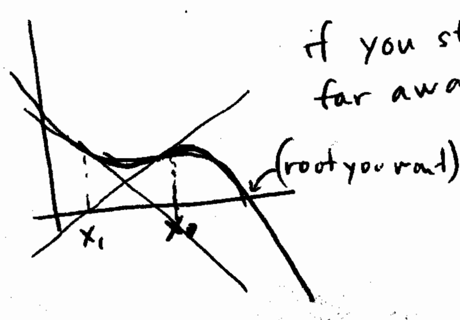
if $f'(x_n)=0$ (tangent line is horizontal)

(2)



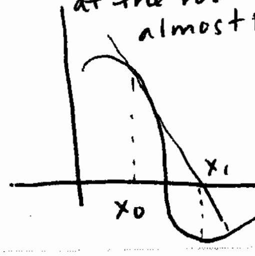
~~if you start too far away~~ if you find the wrong root

(3)

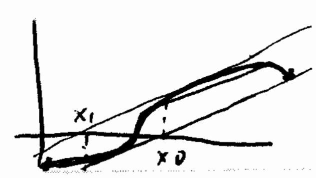


if you start too far away

(4) if the tangent at the root is almost horiz.



(5) if you have 2 tan lines that are para and you enter a "loop of death."



Appendix F Math Xb
April 5, 2004

Common Mistakes

- 1st of all, the most common mistake was inappropriate use of L'Hôpital's Rule.
- L'Hôpital says that $\lim_{x\to a}\frac{f(x)}{g(x)} = \lim_{x\to a}\frac{f'(x)}{g'(x)}$
- we can only use L'Hôpital when $\lim_{x\to a}\frac{f(x)}{g(x)} = \frac{0}{0}$ or $\frac{\infty}{\infty}$.

if it is not in one of these two forms, we ~~have to~~ either just substitute in the value to get an answer, or if we get another indeterminate form like 1^∞ or $0\cdot\infty$, we have to manipulate it until we get it into $\frac{0}{0}$ or $\frac{\infty}{\infty}$ and then use L'Hôp.

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ex: #7

$$\lim_{t\to 0}\frac{t^2+3}{2t^3+100t+1} = \frac{0^2+3}{0+0+1} = 3 \neq \frac{0}{0},\ \neq \frac{\infty}{\infty}$$

so this is our answer, we never had to use L'Hôp.

#6

$$\lim_{x\to\infty}\frac{\ln(5+e^x)}{3x} = \frac{\infty}{\infty} \to \text{L'Hôp} \to \lim_{x\to\infty}\frac{\frac{1}{5+e^x}\cdot e^x}{3} \Rightarrow \lim_{x\to\infty}\frac{e^x}{3(5+e^x)} = \lim_{x\to\infty}\frac{e^x}{15+3e^x} = \frac{\infty}{\infty}$$

→ L'Hôp again → $\lim_{x\to\infty}\frac{e^x}{3e^x}$ → simplify → $\lim_{x\to\infty}\frac{1}{3} = \boxed{\frac{1}{3}}$

#13

$$\lim_{x\to\infty}\frac{e^{2x}+7}{5e^{3x}-10} = \frac{\infty}{\infty}$$ → use L'Hôp.

$$\lim_{x\to\infty}\frac{2e^{2x}}{15e^{3x}}$$ → simplify

$$\lim_{x\to\infty}\frac{2}{15e^x} = \boxed{0}$$

#19

$$\lim_{x\to 0^+} e^x\cdot\ln x = e^{0^+}\cdot\ln 0^+ = 1\cdot -\infty = \boxed{-\infty}$$

→ no need to use L'Hop since we don't have an indet. form.

#20 Remember how we proved that $\lim_{x\to\infty}\left(1+\frac{1}{x}\right)^x = e$? We do the same thing.
→ (this is kind of a weird result, not something we would have expected, huh?)

$$\lim_{x\to\infty}\left(1+\frac{5}{x}\right)^{3x} = L$$ → I set my limit = "L"
→ take the ln of both sides

$$\lim_{x\to\infty}\ln\left(1+\frac{5}{x}\right)^{3x} = \ln L$$

$$\lim_{x\to\infty} 3x\cdot\ln\left(1+\frac{5}{x}\right)$$ → log rules

$$= \infty\cdot\ln 1 = \infty\cdot 0$$ → it is indet. of the form $\infty\cdot 0$ so we have to somehow get it into the form $\frac{0}{0}$ or $\frac{\infty}{\infty}$

$$= \lim_{x\to\infty}\frac{\ln\left(1+\frac{5}{x}\right)}{\frac{1}{3x}} = \frac{0}{0}$$ → now we can use L'Hôpital's Rules

$$= \lim_{x\to\infty}\frac{\frac{1}{\left(1+\frac{5}{x}\right)}\cdot -5x^{-2}}{-3x^{-2}} = \lim_{x\to\infty}\frac{15}{\left(1+\frac{5}{x}\right)}$$ → simplify, denominator → 1

$= 15$ [illegible] → since $15 = \ln L$ $\boxed{L = e^{15}}$!

#22

$f(x) = x^2 \cdot \ln x$

a) to find the min, we take the derivative ~~at~~ set $= 0$:

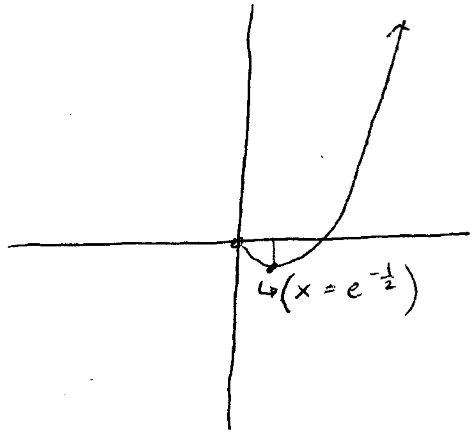
$f'(x) = 2x \cdot \ln x + x^2 \cdot \frac{1}{x}$

$0 = 2x\ln x + x$

$0 = x(2\ln x + 1)$

$x = 0$ or $\ln x = -\frac{1}{2}$

$x = e^{-\frac{1}{2}}$



two crit pts however $f(x)$ is undef. at $x = 0$, judging from the graph, our local min is $x = e^{-\frac{1}{2}}$

b) to find the limit as $x \to 0^+$

$\lim_{x \to 0^+} x^2 \cdot \ln x = 0 \cdot -\infty$

→ indet form of $0 \cdot -\infty$, we have to get it into either $\frac{0}{0}$ or $\frac{\infty}{\infty}$

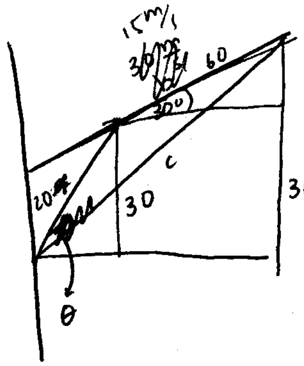
$\lim_{x \to 0^+} \frac{\ln x}{\frac{1}{x^2}} = \lim_{x \to 0} \frac{\frac{1}{x}}{-2x^{-3}}$

→ we have 2 choices: (1) $\lim_{x \to 0} \frac{\ln x}{\frac{1}{x^2}}$

(2) $\lim_{x \to 0} \frac{x^2}{\frac{1}{\ln x}}$

$= \lim_{x \to 0^+} \frac{x^2}{-2} = \boxed{0}$

→ if you try using L'Hôpital's on both you will see that (2) keeps on giving indet. forms. (if you ever get in that situation, just try switching the fraction)



$c^2 = a^2 + b^2 - 2ab\cos C$

$c^2 = 20^2 + b^2 - 40b\cos C$

$2c\frac{dc}{dt} = 2b\frac{db}{dt} - \left(40b(-\sin C)\frac{dC}{dt} + 40\cos C\right)$

$2c\frac{dc}{dt} = 2b(15) +$