Review: Ch. 23-25, 27.2

$\boxed{\S 23.1}$

Defn: The area ftn ${}_aA_f(x)$ gives the signed area btwn the horizontal axis and the ftn $f(t)$ btwn a and x.

$${}_aA_f(x)$$

ending pt

start pt. "area" ftn we want the area of

(*) another way of writing the area is the integral

$${}_aA_f(x) = \int_a^x f(t)\,dt$$

problems: 3, 4

$\boxed{\S 23.2}$

Properties of the area ftn:

- behaves like $f(x)$ is its derivative
- actual y-value at any pt changes; based on what "a" is, the graph of the area ftn is vertically shifted (up or down)

problems: 1

$\boxed{\S 23.3}$

Fundamental Thm of Calculus, Version 1:

$${}_cA_f(x) = \int_c^x f(t)\,dt \quad \text{for } x \in [a,b]$$

if f is continuous and $c \in [a,b]$

This means that:

$$\frac{d}{dx}\int_a^x f(t)\,dt = f(x)$$

problems: 4 (also, review Hw on this section)

$\boxed{§\,24.1}$

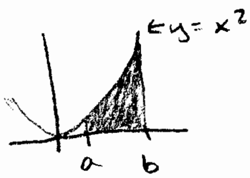
Defn: F is an antiderivative of f if $F' = f$.

Fundamental Thm of Calc, V2:

$$\int_a^b f(t)\,dt = F(b) - F(a)$$

if $F' = f$ and f is continuous on $[a, b]$

Graphically:



Shaded Area =

$$\int_a^b x^2\,dx = \frac{x^3}{3}\Big|_a^b = \frac{b^3}{3} - \frac{a^3}{3}$$

↑ antiderivative of x^2

problems: 3, 4, 14, 16, 21, 22
challenge: 17, 18

$\boxed{§\,24.2}$

Average Value of $f(x)$ on $[a, b]$ is:

$$\frac{1}{b-a}\int_a^b f(x)\,dx$$

(*) pay attention to signed area

note: read "some common misconceptions" on p777.

problems: 7, 8, 11

$\boxed{§\,25.1}$

Indefinite Integral:

$\int f(x)\,dx$ [The "integrand" is what you're integrating. In this case, it's $f(x)$]

Recall that $F(x)$ is the antiderivative of $f(x)$.

$\int f(x)\,dx = F(x) + C \Rightarrow$ Please remember the "+C"!

(*) To check what you got for an integral, take the derivative of your answer! It should be what you started with.

(*) Note relation btwn "integral" and "area from ftn to horizontal axis"

§ 25.1 Cont'd

List of handy integrals

$\int x^n \, dx = \frac{x^{n+1}}{n+1} + C$ for $n \neq -1$

$\int \frac{1}{x} dx = \int x^{-1} dx = \ln|x| + C$

$\int e^x \, dx = e^x + C$

$\int b^x \, dx = \frac{1}{\ln b} b^x + C$

$\int \cos x \, dx = \sin x + C$

$\int \sin x \, dx = -\cos x + C$

$\int \sec^2 x \, dx = \tan x + C$

$\int \frac{1}{\sqrt{1-x^2}} dx = \sin^{-1} x + C$

$\int \frac{-1}{\sqrt{1-x^2}} dx = \cos^{-1} x + C$

$\int \frac{1}{1+x^2} dx = \tan^{-1} x + C$

problems: 4, 8, 18

§ 25.2

If there's a seemingly impossible integral,

eg. $\int \frac{3x}{1+x^2} dx$

try to see if you can do a U substitution

eg. let $u = 1+x^2$

$du = 2x \, dx$ derivative of above relationship

So that your integral becomes

$$3 \int \frac{\frac{1}{2} du}{u} = \frac{3}{2} \int \frac{1}{u} du$$

(notice that if you plugged back in "$2x\,dx$" for du and "$1+x^2$" for u, you would have your starting integral)

Once we subbed in u, we can:

$\frac{3}{2} \int \frac{1}{u} du = \frac{3}{2} \ln|u| + C = \frac{3}{2} \ln|1+x^2| + C$ if you sub u in, then sub back in for u at the end!

If the integral was definite

eg. $\int_a^b \frac{3x}{1+x^2} dx$

make sure you change a & b into their u equivalents. Once you've substituted EVERYTHING to u's, you can solve through w/o having to sub back in for u at the end.

§ 25.2 Cont'd

Key: look for a function AND its derivative inside the integral.

problems: Do as many practice problems as you can here!

1, 3–7, 15–23, 25

§ 25.3

Sometimes, U sub. can be used to make an integral easier

eg $\int \frac{x}{x+2}\,dx$

let $u = x+2$

$du = dx$

- we know that $x = u-2$, so we plug "$u-2$" in whereever we see x

$$\int \frac{x}{x+2}\,dx \Rightarrow \int \frac{u-2}{(u-2)+2}\,du = \int \frac{u-2}{u}\,du$$

- we can simplify this fraction now!

$$\int \frac{u-2}{u}\,du = \int 1 - \frac{2}{u}\,du$$

- solve

$$\int 1 - \frac{2}{u}\,du = u - 2\ln|u| + C$$

- sub u out (since we want our answers in terms of x)

$$u - 2\ln|u| + C = (x+2) - 2\ln|x+2| + C$$

★ See example 25.7 on p. 800–802

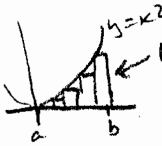
problems: 2, 5, 6, 14

27.2

To find the area of a region bounded by 2 or more curves:

- Think of the "$\int_a^b$" to mean "add all slices btwn a and b"
- Graph & shade in the region
- figure out the height of each slice (and whether slices go horizontally or vertically)

What is a "slice"?



When we went from the estimate to the exact area, we did so by making these rectangles really skinny, so that we get an infinite number of slices.

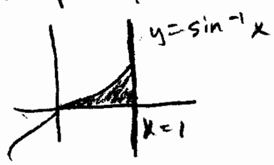
Imagine this is a "slice" from the above graph. What's its area? Its width is dx, and its height (in this case) is f(x), so its area is $f(x) \cdot dx$

To add up all the slices, we just add their areas from a to b.

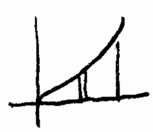
$\int_a^b f(x)\,dx$ ← this expresses the area of your region.

eg. find area btwn $y=\sin^{-1}x$, line $x=1$, and x-axis

1) Graph & Shade!



2) If we set this up w/ vertical slices:



our slice's area would be:

$\sin^{-1}x\,dx$

So our integral is $\int_0^1 \sin^{-1}x\,dx$

BUT we don't know the antiderivative of that!

Slicing horizontally:

The width of a slice would be dy, and the height would be $1-\sin y$ (it's 1 − the x value, and since I want everything to use the same variable, I express x as $\sin y$, which I got by using $y=\sin^{-1}x$)

So our integral is:

$\int_0^{\pi/4} (1-\sin y)\,dy$ → now just solve this definite integral!

↑ note that these are y-values!

[illegible]ms: 4, 7, 8, 11, 14