Math Xb Spring 2004
Final Exam Review Guide

1 Topics

The final exam will cover chapters 17 through 25 (omitting §18.3 and §20.7), as well as §27.2, §15.2, §31.1, and Appendices F and G in our textbook, *Calculus: An Integrated Approach to Functions and Their Rates of Change*. In particular, you will be responsible for the following topics.

Chapter 17: Implicit Differentiation and Its Applications

- To use logarithmic differentiation to find the derivatives of functions of the form $f(x)^{g(x)}$
- To use implicit differentiation to find $\frac{dy}{dx}$ given an equation involving x and y
- To find the slope of a tangent line to a given curve (described by an equation involving x and y) at a given point
- To find the points on a curve (described by an equation involving x and y) at which the tangent line has a given slope
- To understand how to use a geometric relationship between two or more variables that depend on time to find a relationship between the rates of change of those variables

Chapter 18: Geometric Sums and Series

- To recognize a finite geometric sum and identify its common ratio.
- To express a geometric sum in closed form.
- To compute a numeric geometric sum.
- To determine if a given geometric series converges or diverges.
- To find the sum of a given convergent geometric series.
- To express and analyze a geometric sum or series using summation notation.
- To be able to use geometric sums and series to solve problems in a variety of contexts.

Chapters 19 & 20: Trigonometry

- To understand sine and cosine as functions of arc length on the unit circle.
- To approximate sine, cosine, and tangent values given a calibrated unit circle.
- To be familiar with the graphs of the sine, cosine, and tangent functions.
- To understand the periodicity of the sine and cosine functions.
- To identify the balance value, amplitude, and period of a sinusoidal function given its formula or graph.
- To use trig functions to model other functions.
- To understand the interpretation of $\tan x$ as the slope of a certain line.
- To understand the relationship between angles and arc length.

- To take advantage of circle symmetry when finding trig function values.
- To understand the relationship between sine, cosine, and tangent and right triangles.
- To know the sine, cosine, and tangent values of $\frac{\pi}{6}$, $\frac{\pi}{4}$, and $\frac{\pi}{3}$.
- To "solve" triangles, that is, to determine all angles and sides of a triangle from some given information.
- To understand the inverse trig functions $\sin^{-1}$, $\cos^{-1}$, and $\tan^{-1}$ and their domains and ranges.
- To simplify expressions involving inverse trig functions by using triangles.
- To solve equations involving trig functions on both restricted and unrestricted domains.
- To be able to apply the Law of Cosines and the Law of Sines.
- To know the identities listed on the Trig Identities handout from March 10th.

Chapter 21: Differentiation of Trig Functions

- To know the derivatives of the six trigonometric functions.
- To find derivatives of more complex functions that involve trigonometric functions.
- To solve related rates problems involving trig functions.
- To solve optimization problems involving trig functions.
- To solve curve-sketching problems involving trig functions.
- To know how one can find the derivatives of the inverse trig functions using implicit differentiation.
- To know the derivatives of $\sin^{-1}$, $\tan^{-1}$, and $\cos^{-1}$.

Appendices F and G: L'Hôpital's Rule and Newton's Method

- To recognize and evaluate the indeterminate forms $\frac{0}{0}$, $\frac{\infty}{\infty}$, $0\cdot\infty$, 1^∞, ∞^0, and 0^0 using L'Hôpital's Rule.
- To understand how Newton's Method works and how it can go wrong.
- To approximate roots of functions iteratively using Newton's Method.

Chapter 22: Net Change, Area, and the Definite Integral

- To understand the relationship between the definite integral and the questions
  1. Given a rate function, how do we calculate the net change in amount?
  2. How do we calculate the signed area between the graph of a function and the horizontal axis?

  and to use these relationships to solve problems (such as §22.1 #1, §22.2 #5, and §22.3 #3).
- To approximate a definite integral (or net change or signed area) using left- and right-hand sums.
- To approximate the net change in amount given sample points on a rate function using left- and right-hand sums.
- To evaluate a definite integral (or net change or signed area) using geometric area formulas.
- To understand the definition of the definite integral.
- To use properties of definite integrals to assist in simplifying definite integrals.

Chapters 23 & 24: The Area Function and the FTC

- To find the area function ${}_cA_f$ for a function f.

- To interpret the area function as a net change function.
- To know why two area functions ${}_cA_f$ and ${}_dA_f$ differ by a constant.
- To determine where an area function A_f is increasing, decreasing, concave up, and concave down by examining f.
- To know the Fundamental Theorem of Calculus (Versions 1 and 2).
- To apply the Fundamental Theorem of Calculus to evaluate definite integrals.
- To use properties of definite integrals to assist in evaluating definite integrals.
- To compute the average value of a function on a given interval.
- To interpret the average value of a function in a particular context (e.g., velocity or speed).

Chapter 25 & §27.2: Applications and Computation of the Integral

- To evaluate simple definite and indefinite integrals. (See the list on page 784.)
- To evaluate more complex definite and indefinite integrals using algebra and/or the substitution rule.
- To understand how the area between two curves can be thought of as a Reimann sum.
- To find the area between two curves by integrating with respect to x or y.
- To find the area between two curves when $f(x) \geq g(x)$ for some values of x and $g(x) \geq f(x)$ for other values of x.

§15.2 & §31.1: Differential Equations

- To use the differential equation $\frac{dy}{dt} = ky$ and its solution $y(t) = Ce^{kt}$ to model and analyze exponential growth and decay problems.
- To solve the differential equation $\frac{dy}{dt} = ky$ given an initial condition.
- To interpret a given differential equation in the context of a particular application.
- To write a differential equation which models a given situation.

2 Suggested Exercises

Chapter 17

- §17.1 #4
- §17.2 #1, 5
- §17.3 #2, 4, 5, 7, 10
- §17.4 #1, 6, 8, 10, 13

Chapter 18

- §18.1 #1, 4, 11, 12, 18, 24, 27, 32
- §18.2 #3, 4, 6, 8, 11
- §18.4 #7, 8, 11, 20
- §18.5 #6, 8, 12, 13, 26

Chapters 19 & 20

- §19.1 #1, 2, 5
- §19.2 #4, 5, 7, 14, 15
- §19.3 #2, 4, 6, 12
- §19.4 #1, 5, 8, 10
- §20.1 #2, 3, 5
- §20.2 #2, 5, 7, 9
- §20.3 #4, 5, 8, 9, 10, 12, 13
- §20.4 #3, 4, 9, 10, 12, 22
- §20.5 #5, 6
- §20.6 #4, 5, 6, 9

Chapter 21

- §21.2 #4, 5, 7, 9, 15
- §21.3 #2, 6, 13, 15, 17 (a)
- §21.4 #6, 8, 9

Appendices F and G

- App. F #1, 2, 5, 12, 13, 18
- App. G #2, 4, 8

Chapter 22

- §22.1 #2, 8, 9
- §22.2 #7, 8
- §22.3 #2, 4, 7
- §22.4 #4, 7, 9

Chapters 23 & 24

- §23.1 #1, 3
- §23.2 #3
- §23.3 #4
- §24.1 #7, 10, 11, 12, 19
- §24.2 #3, 4, 6, 7, 8

Chapter 25 & §27.2

- §25.1 #2–10 even, 16, 18
- §25.2 #4, 5, 6, 9, 25
- §25.3 #2–10 even
- §27.2 #2, 4, 7, 8, 10, 12

§15.2 & §31.1

- §15.2 #4, 8, 9, 12
- §31.1 #5, 6