

Differential Equation - an equation that has a derivative

in it (ex) $\frac{dy}{dt} = y$, $\frac{ds}{dx} = 2s$, $(\frac{dp}{dt})^2 + \frac{dp}{dt} + 4 = 0$

(just an example you do NOT know how to solve this one)

• Exponential functions are very useful in solving differential equations because if: $y = e^x$
 $\frac{dy}{dx} = e^x$

so $\frac{dy}{dx} = y$ like the 1st example

• also if $y = e^{2x}$, $\frac{dy}{dx} = 2e^{2x}$ or $\frac{dy}{dx} = 2y$ like the 2nd example

* In general if $\frac{dy}{dx} = Cy$
 $y = Ke^{Cx}$ where K & C are constants

You can memorize this or derive it

(ex) $\frac{dy}{dx} = Cy$ (try to get all y's on one side & x's on the other)

$\frac{1}{y} dy = C dx$ - this looks sort of like an integral

$\int \frac{1}{y} dy = \int C dx$ - now integrate both sides

$\ln y + a = Cx + b$ these are both constants

$\ln y = Cx + (b+a) = Cx + d$

$e^{\ln y} = y = e^{Cx+d} = e^{Cx} (e^d)$ these are both constants

$y = Ke^{Cx}$

↑ This is a solution to $\frac{dy}{dx} = Cy$

A Solution of a differential equation is a function $f(x)$ whose derivative fits the original differential equation

(ex) $y = Ke^{Cx}$ is a solution of $\frac{dy}{dx} = Cy$
 because $\frac{dy}{dx} = CKe^{Cx} = Cy$

Q. Are the following equations solutions to $\frac{dy}{dx} = \frac{1}{2}y$

(a) $y = \frac{1}{2}$

(c) $y = \frac{1}{2}e^x$

(e) $y = e^{\frac{x}{2}}$

(b) $y = 4e^{\frac{x}{2}}$

(d) $y = e^{\frac{x}{2}}$

(f) $y = \frac{1}{2} \ln|x|$

A. (a) $\frac{dy}{dx} = 0$ NO

(d) $\frac{dy}{dx} = \frac{1}{2}e^{\frac{x}{2}}$, YES

(b) $\frac{dy}{dx} = \frac{1}{2}(4e^{\frac{x}{2}}) = \frac{1}{2}y$, YES

(e) $\frac{dy}{dx} = 0$ (no x in function) NO

(c) $\frac{dy}{dx} = \frac{1}{2}e^x = y$, NO

(f) $\frac{dy}{dx} = \frac{1}{2x}$ NO

General Solution - a solution which contains an undetermined constant
 (ex) $y = ke^t$, $y = e^x + B$, $P = x^2 + Cx$

Particular Solution - a solution for which the value of the constant has been determined using a known value of the function.
 (ex) $y = 5e^t$, $y = e^x + 100$, $P = x^2 + 2x$

Q. Find the particular solution:

(a) $f(x) = Ce^{2x}$; $f(0) = 400$

(b) $f(r) = Ce^{\frac{1}{2}r}$; $f(2) = e$

(c) $f(t) = Ce^{4t}$; $f(3) = 1$

A. (a) $f(0) = Ce^{2 \cdot 0} = C(1) = 400$

$C = 400$

$f(x) = 400e^{2x}$

(c) $f(3) = Ce^{4 \cdot 3} = Ce^{12} = 1$

$C = e^{-12}$

$f(t) = e^{-12}e^{4t} = e^{4t-12}$

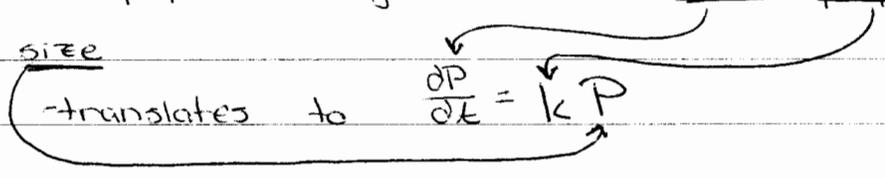
(b) $f(2) = Ce^{\frac{1}{2} \cdot 2} = Ce = e$

$C = 1$

$f(r) = e^{\frac{1}{2}r}$

- Differential Equations can be used to model many situations in Biology, Economics, chemistry, etc.
- some connections to make in work problems
- rate = a derivative, decay/decrease = -rate, grows/increases = +rate
- proportional = a proportionality constant
- value, size, population etc. = the function that is the solution to the differential equation

ex. A population grows at a rate proportional to its own



Q. A new element, Mathium, decays at a rate proportional to $\frac{1}{10}$ of the amount of Mathium present

- translate this to an equation
- suppose that the statement above still holds, but 10 grams of Mathium is added to the amount present continuously each year. Translate this new situation to an equation.

A. (a) $\frac{dM}{dt} = -\frac{1}{10}M$
 (b) $\frac{dM}{dt} = -\frac{1}{10}M + 10$