<u>Differential Equation</u> - an equation that has a derivative in it (ex) $\frac{dy}{dt} = y$, $\frac{ds}{dx} = 2s$, $\left(\frac{dp}{dt}\right)^2 + \frac{dp}{dt} + 4 = 0$ (just an example you do <u>NOT</u> know how to solve this one)

- Exponential functions are very useful in solving differential equations because if: $y = e^x$

$$\frac{dy}{dx} = e^x$$

so $\frac{dy}{dx} = y$ like the 1st example

- also if $y = e^{2x}$, $\frac{dy}{dx} = 2e^{2x}$ & $\frac{dy}{dx} = 2y$ like the 2nd example

* <u>In general</u> if $\frac{dy}{dx} = Cy$

$$y = Ke^{ct}$$ where K & C are constants

You can memorize this or derive it

(ex) $\frac{dy}{dx} = Cy$ (try to get all y's on one side & x's on the other)

$\frac{1}{y}\,dy = C\,dx$ - this look sort of like an integral

$\int \frac{1}{y}\,dy = \int C\,dx$ - so integrate both sides

$\ln y + a = Cx + b$ — these are both constants

$\ln y = Cx + (b+a) = Cx + d$

$e^{\ln y} = y = e^{Cx+d} = e^{Cx} e^{d}$ — these are both constants

$y = Ke^{Cx}$

↑ This is a solution to $\frac{dy}{dx} = Cy$

A <u>Solution</u> of a differential equation is a function f(x) whose derivative fits the original differential equation

(ex) $y = Ke^{Cx}$ is a solution of $\frac{dy}{dx} = Cy$

because $\frac{dy}{dx} = CKe^{Cx} = Cy$

Q. Are the following equations solutions to $\frac{dy}{dx} = \frac{1}{2}y$

(a) $y = \frac{1}{2}$ (c) $y = \frac{1}{2}e^{x}$ (e) $y = e^{\frac{t}{2}}$

(b) $y = 4e^{\frac{x}{2}}$ (d) $y = e^{\frac{x}{2}}$ (f) $y = \frac{1}{2}\ln|x|$

A. (a) $\frac{dy}{dx} = 0$ NO

(b) $\frac{dy}{dx} = \frac{1}{2}(4e^{\frac{x}{2}}) = \frac{1}{2}y$, yes

(c) $\frac{dy}{dx} = \frac{1}{2}e^{x} = y$, NO

(d) $\frac{dy}{dx} = \frac{1}{2}e^{\frac{x}{2}}$, yes

(e) $\frac{dy}{dx} = 0$ (no x in function) NO

(f) $\frac{dy}{dx} = \frac{1}{2x}$ NO

General Solution - a solution which contains an undetermined constant ex $y = Ke^{t}$, $y = e^{x} + B$, $P = x^2 + Cx$

Particular Solution - a solution for which the value of the constant has been determined using a known value of the function. ex $y = 5e^{t}$, $y = e^{x} + 100$, $P = x^2 + 2x$

Q. Find the particular solution:

(a) $f(x) = Ce^{2x}$; $f(0) = 400$

(b) $f(r) = Ce^{\frac{1}{2}r}$; $f(2) = e$

(c) $f(t) = Ce^{4t}$; $f(3) = 1$

A. (a) $f(0) = Ce^{2 \cdot 0} = C(1) = 400$

$C = 400$

$\boxed{f(x) = 400e^{2x}}$

(b) $f(2) = Ce^{\frac{1}{2}\cdot 2} = Ce = e$

$C = 1$

$\boxed{f(r) = e^{\frac{1}{2}r}}$

(c) $f(3) = Ce^{4 \cdot 3} = Ce^{12} = 1$

$C = e^{-12}$

$\boxed{f(t) = e^{-12}e^{4t} = e^{4t-12}}$

- Differential Equations can be used to model many situations in Biology, economics, chemistry, etc.
- some connections to make in work problems
- rate = a derivative, decay / decrease = − rate, grows / increases = + rate
- proportional = a proportionality constant
- value, size, population etc. = the function that is the solution to the differential equation

ex. A population grows at a rate proportional to its own size

translates to $\frac{dP}{dt} = kP$

Q. A new element, Mathium, decays at a rate proportional to $\frac{1}{10}$ of the amount of Mathium present

(a) translate this to an equation

(b) suppose that the statement above still holds, but 10 grams of Mathium is added to the amount present continuously each year. Translate this new situation to an equation.

A. (a) $\frac{dM}{dt} = \frac{-1}{10} M$

(b) $\frac{dM}{dt} = \frac{-1}{10} M + 10$