# Modeling With Differential Equations

A basic principle

total rate of change = birth rate − death rate

= amount of increase − amount of decrease

= amount in − amount out

= etc.

(Ex) In a population of bacteria the rate of population increase (per day) is proportional to 4 times the size of the population, additionaly 100 bacteria die each day regardles of (time) population size. What is the total rate of change of population?

$$-\frac{dP}{dt} = 4P - 100$$

total change = birth rate − death rate

- Note: this situation is often tricky & many people write the equation as $\frac{dP}{dt} = 4P - 100t$, but $100t$ is how many bacteria die in the first $t$ days. Not how many die each day

**Q.** Translate the following into an equation:

Each day you take 30mg of B12. B12 is cleared from the body at a rate proportional to the amount in the blood stream. How is the amount of B12 changing in the blood stream?

**A.** $\frac{dB}{dt} = 30 - kB$

• Other situations may depend on a difference or product of quantities.

Q. Write an equation to describe the following:

(a) In a non-reproducing population of [redacted] mice, the ~~change~~ decrease in population is proportional to the product of the size of the mouse population and the number of owls, O.

(b) A group of 250 tourists are trapped on a deserted island when a flu epidemic breaks out among them. The rate at which people become ill is proportional to the product of sick people(s) & healthy people.

(c) The rate at which icecream warms (& thus melts) is proportional to the difference between the temperature of the room(R) & the temperature of the ice cream. (I)

A. (a) $\frac{dm}{dt} = 0 - k(mC)$

(b) $\frac{ds}{dt} = k(s)(250-s)$

(c) $\frac{dI}{dt} = k(R-I)$