Math Xb Spring 2004 Name ________________

Lab 1: Exponential and Logarithmic Functions Section Instructor ________________

February 12, 2004 Collaborators ________________

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Certain combinations of the exponential functions e^x and e^{-x} arise so frequently in mathematics and its applications that they are given special names. Two of them are the *hyperbolic sine function*, denoted $\sinh x$ (pronounced "sinch x"), and the *hyperbolic cosine function*, denoted $\cosh x$ (pronounced "cosh x"). They are defined as follows.

$$\sinh x = \frac{e^x - e^{-x}}{2}$$

$$\cosh x = \frac{e^x + e^{-x}}{2}$$

These functions are similar in some ways to the trigonometric functions we will study later this semester. You need not know any trigonometry to complete this lab.

1. Choose one of the two hyperbolic trig functions listed above. Explain how to use the graph of $y = e^x$ to construct the graph of your chosen function without using a graphing calculator. (Use what you know about stretching, shrinking, shifting, flipping, adding, and subtracting the graphs of functions.) Illustrate your explanation with appropriate graphs.

2. Simplify $\cosh^2 x - \sinh^2 x$.

3. The graph of $y = \sinh x$ passes the vertical line test, and so $\sinh x$ is a one-to-one, and hence invertible, function. The inverse of $\sinh x$ is the function $\sinh^{-1} x = \ln(x + \sqrt{x^2 + 1})$. Find and simplify the derivative of $\sinh^{-1} x$.

4. It can be shown that if a heavy flexible cable (such as a telephone or power line) is suspended between two points at the same height, then it takes the shape of a curve with equation $y = c + a\cosh(\frac{x}{a})$ called a *catenary*. (The Latin word *catena* means "chain.") Suppose that a telephone line hangs between two poles 14 meters apart in the shape of the catenary $y = 20\cosh(\frac{x}{20}) - 15$, where x and y are measured in meters. Find the slope of this curve where it meets the right pole.