Math Xb Spring 2004

Lab 6: An Interesting Limit

March 23, 2004

Name _____

Section Instructor _____

Collaborators _____

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1. Find each of the following limits.

    (a) $\displaystyle\lim_{x\to\infty}\left(1+\frac{1}{10}\right)^{x}$

    (b) $\displaystyle\lim_{x\to\infty}\left(1+\frac{1}{100}\right)^{x}$

    (c) $\displaystyle\lim_{x\to\infty}\left(1+\frac{1}{1000}\right)^{x}$

    (d) $\displaystyle\lim_{x\to\infty}\left(1+\frac{1}{10000}\right)^{x}$

2. Based on your answers to Question 1, what does $\displaystyle\lim_{x\to\infty}\left(1+\frac{1}{x}\right)^{x}$ appear to be?

3. Find each of the following limits.

    (a) $\displaystyle\lim_{x\to\infty}\left(1+\frac{1}{x}\right)^{10}$

    (b) $\displaystyle\lim_{x\to\infty}\left(1+\frac{1}{x}\right)^{100}$

    (c) $\displaystyle\lim_{x\to\infty}\left(1+\frac{1}{x}\right)^{1000}$

    (d) $\displaystyle\lim_{x\to\infty}\left(1+\frac{1}{x}\right)^{10000}$

4. Based on your answers to Question 3, what does $\displaystyle\lim_{x\to\infty}\left(1+\frac{1}{x}\right)^{x}$ appear to be?

5. The expression $\displaystyle\lim_{x\to\infty}\left(1+\frac{1}{x}\right)^{x}$ is said to be of indeterminate form 1^{∞}. Given your answers to the previous questions, explain in words why something of the form 1^{∞} is indeterminate.

6. Graph the function $f(x) = \left(1 + \dfrac{1}{x}\right)^x$ on your graphing calculator on the domain $[0, 100]$.

   (a) Sketch the graph in the space below. Make clear any important features of the graph.

   (b) Does $\displaystyle \lim_{x \to \infty} \left(1 + \dfrac{1}{x}\right)^x$ appear to exist?

7. Use your calculator to complete the following table of values for $f(x) = \left(1 + \dfrac{1}{x}\right)^x$. Give your answers in decimal form, accurate to eight decimal places.

x	$f(x)$
10	
100	
1,000	
10,000	
100,000	

8. Given your answers to the previous two questions, $\displaystyle \lim_{x \to \infty} \left(1 + \dfrac{1}{x}\right)^x$ is probably equal to what popular mathematical constant?

9. Does the series $\displaystyle \sum_{n=1}^{\infty} \left(1 + \dfrac{1}{n}\right)^n$ converge or diverge? Explain the reasoning behind your answer. (Note that the series is *not* geometric, but we can still determine if it converges or diverges.)