

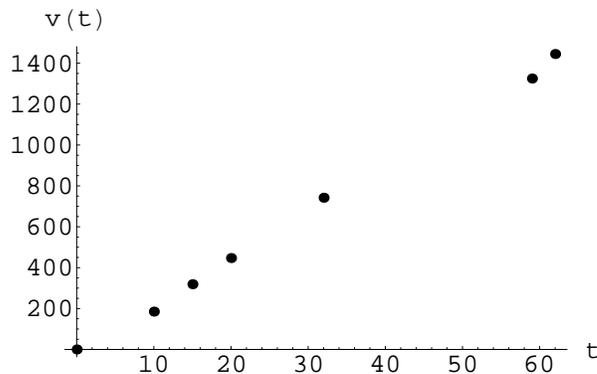
When we estimate distances from velocity data it is sometimes necessary to use times $t_0, t_1, t_2, t_3, \dots$ that are not equally spaced. For example, on May 7, 1992, the space shuttle *Endeavor* was launched on mission STS-49, the purpose of which was to install a new perigee kick motor in an Intelsat communications satellite. The table below gives the velocity data for the shuttle between liftoff and the jettisoning of the solid rocket boosters.

Event	Times (s)	Velocity (ft/s)
Launch	0	0
Begin roll maneuver	10	185
End roll maneuver	15	319
Throttle to 89%	20	447
Throttle to 67%	32	742
Throttle to 104%	59	1325
Maximum dynamic pressure	62	1445
Solid rocket booster separation	125	4151

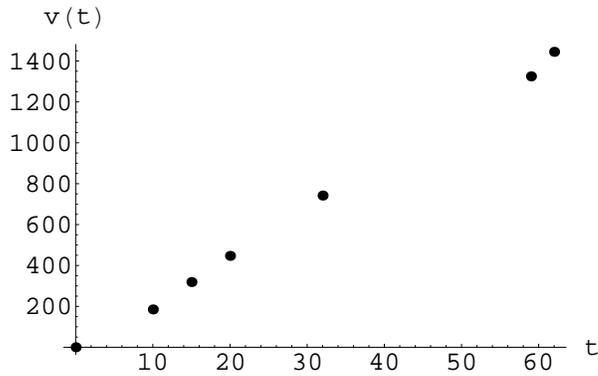
These data can be used to give an estimate of the height of the shuttle after liftoff. (Assume that the velocity of the shuttle is increasing during the first 125 seconds of flight.)

1. If you use a left-hand sum to approximate the height of the shuttle at some time during the first 125 seconds of flight, will you get a lower bound or an upper bound? Explain why.

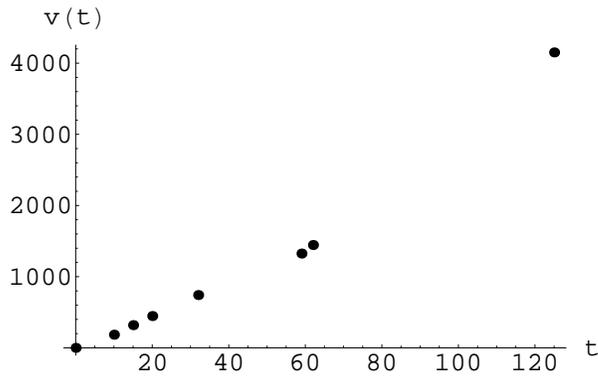
2. Use the data above to give a lower bound on the height of the shuttle 62 seconds after liftoff. Illustrate your answer on the graph below.



3. Use the data above to give an upper bound on the height of the shuttle 62 seconds after liftoff. Illustrate your answer on the graph below.



4. Use the data above to give an estimate of the height of the shuttle 125 seconds after liftoff. (You may choose whether to find a lower bound or an upper bound.) Illustrate your answer on the graph below.



5. Do you think that your estimate in Question 4 is a good estimate or a poor estimate? Explain why.

6. How might you improve the estimates you found above? Explain what you might do and how it would help.