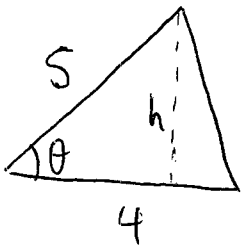
Worksheet 3/15

2.



$\frac{d\theta}{dt} = .006$ radians/sec

$A = \frac{1}{2} \cdot \text{base} \cdot \text{height} = \frac{1}{2} \cdot 4 \cdot h$

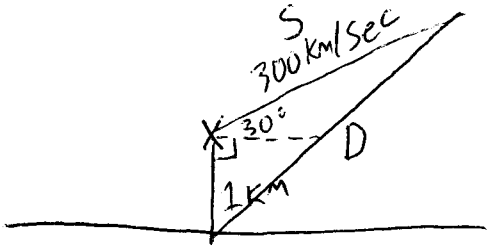
$\sin\theta = \frac{h}{5} \Rightarrow h = 5\sin\theta$. So $A = \frac{1}{2} \cdot 4 \cdot 5\sin\theta$

$A = 10\sin\theta$

$\frac{dA}{dt} = 10\cos\theta \cdot \frac{d\theta}{dt} = 10 \cdot \cos\theta \cdot .006$ When $\theta = \pi/3$, $\cos\theta = \frac{1}{2}$.

So $\frac{dA}{dt} = .03$ meters^2/sec

3.



use law of cosines:

$D^2 = 1^2 + S^2 - 2 \cdot 1 \cdot S \cdot \cos 120$

$= 1 + S + S^2$

take deriv:

$2D \cdot \frac{dD}{dt} = \frac{dS}{dt} + 2S \cdot \frac{dS}{dt}$

$\frac{dS}{dt} = 300$ km/hour

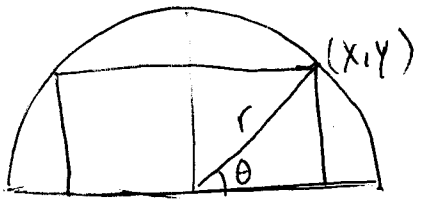
$S(\text{at } t = 1 \text{ min}) = \frac{300}{60}$ km $= 5$ km

$D = \sqrt{1+5+25} = \sqrt{31}$

$\frac{dD}{dt} = \frac{300 + 10 \cdot 300}{2\sqrt{31}}$

≈ 296.3 km/hr

5.



$x = r\cos\theta$
$y = r\sin\theta$

$\text{Area} = 2 \cdot y \cdot x = 2r^2\cos\theta\sin\theta$

$\frac{dA}{d\theta} = 2r^2\cos^2\theta - 2r^2\sin^2\theta$

$0 = 2r^2(\cos^2\theta - \sin^2\theta)$

$\Rightarrow \cos^2\theta = \sin^2\theta \Rightarrow \theta = \pi/4$

then $A = 2r^2 \cdot \frac{\sqrt{2}}{2} \cdot \frac{\sqrt{2}}{2} = \boxed{r^2}$

6. $f(\theta) = \sin^2\theta$, $0 \le \theta \le 2\pi$

a) $f'(\theta) = 2\sin\theta\cos\theta$

increases when $0 < \theta < \pi/2$, $\pi < \theta < 3\pi/2$

decreases when $\pi/2 < \theta < \pi$, $\frac{3\pi}{2} < \theta < 2\pi$

b) local max/min at $\sin\theta$ or $\cos\theta = 0$

$\theta = 0, \pi/2, \pi, 3\pi/2$

$f(0) = 0$ (loc min), $f(\pi/2) = 1$ (loc max)

$f(\pi) = 0$ (loc min), $f(3\pi/2) = 1$ (loc max)

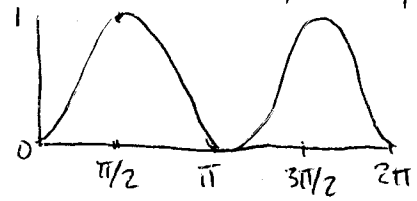
c) $f''(\theta) = 2(\cos^2\theta - \sin^2\theta)$

concave up when $|\cos\theta| > |\sin\theta|$: $0 < \theta < \pi/4$, $\frac{3\pi}{4} < \theta < \frac{5\pi}{4}$, $\frac{7\pi}{4} < \theta < 2\pi$

concave down for all other values of θ btwn 0 and 2π, except inflection points:

d) $\pi/4, 3\pi/4, 5\pi/4, 7\pi/4$ inflection pts

e)



7. $f(t) = t + \cos t$

a) $f'(t) = 1 - \sin t$

increasing except when $\sin t = 1$ $(t = \pi/2, -3\pi/2)$

b) no local max/min

c) $f''(t) = -\cos t \Rightarrow$ concave up when $\cos t < 0$ $\left(-\frac{3\pi}{2} < t < -\frac{\pi}{2}, \frac{\pi}{2} < t < \frac{3\pi}{2}\right)$

concave down everywhere else, except inflection points:

d) $-\cos t = 0 \Rightarrow t = -\frac{3\pi}{2}, -\frac{\pi}{2}, \frac{\pi}{2}, \frac{3\pi}{2}$

inflection pts

e)