Worksheet · The Definite Integral (4/9/04)

① (a) around 6 am   (when p(t) greatest)

(b) around noon (when r(t) least)
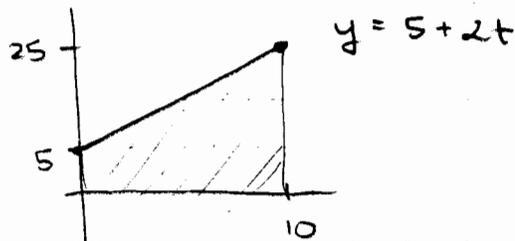
(c) $\int_5^{20} p(t)\,dt - \int_5^{20} r(t)\,dt$

(d) when $\underset{\underset{\substack{\text{amount}\\ \text{produced}\\ \text{in one}\\ \text{24-hr period}}}{\uparrow}}{\int_x^{x+24} p(t)\,dx} - \underset{\underset{\substack{\text{amount}\\ \text{removed}\\ \text{in one}\\ \text{24-hr period}}}{\uparrow}}{\int_x^{x+24} r(t)\,dt}$ is greatest

⟹ around 3 pm

(e) when that quantity ↗ is least ⟹ around 3 am

② (a) $\int_0^{10} (5+2t)\,dt$

y = 5 + 2t

area of trapezoid $= \frac{1}{2}(b_1+b_2)h = \frac{1}{2}(5+25)10 = \boxed{150 \text{ mill.}}$

(b) $\int_0^T (5+2t)\,dt = 14$
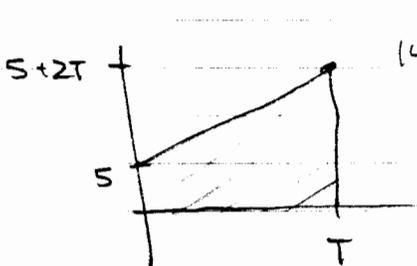
$14 = \int_0^T (5+2t)\,dt = \frac{1}{2}(5+5+2T)\cdot T$
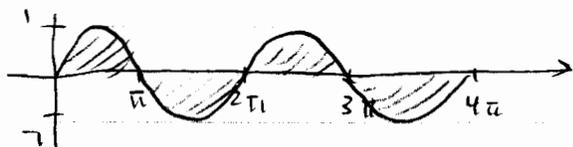
$14 = \frac{1}{2}(10+2T)T$

$14 = (5+T)T$

$14 = 5T + T^2$

$0 = T^2 + 5T - 14 = (T+7)(T-2)$

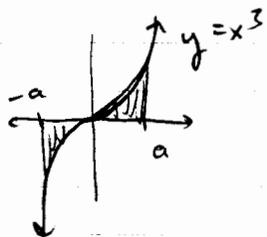$T = -7 \text{ or } 2$

⟹ $\boxed{T = 2 \text{ yrs}}$

③ (a) $\int_0^{4\pi} \sin x \, dx = 0$ since positive and negative area are equal

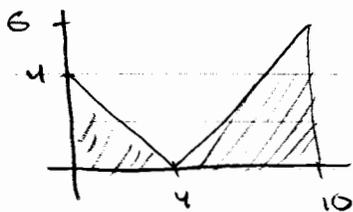(b) $\int_{-a}^{a} x^3 \, dx = 0$ since x^3 is an odd function

(c) $\int_{-5}^{5} \frac{x^2 - 2x^4}{x^3 + x} \, dx = 0$
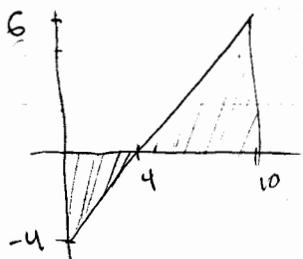
$f(x) = \frac{x^2 - 2x^4}{x^3 + x}$ $f(-x) = \frac{(-x)^2 - 2(-x)^4}{(-x)^3 + (-x)} = \frac{x^2 - 2x^4}{-x^3 - x}$

$= \frac{x^2 - 2x^4}{-(x^3 + x)} = -f(x) \implies$ odd function

(d) $\int_0^{10} |x - 4| \, dx = \frac{1}{2}(4)(4) + \frac{1}{2}(6)(6) = 8 + 18 = \boxed{26}$

(e) $\int_0^{10} (|x| - 4) \, dx = \frac{1}{2}(6)(6) - \frac{1}{2}(4)(4) = 18 - 8 = \boxed{10}$

④ $b < e < c < f < a < b$

$\underbrace{\quad}_{negative} \underset{zero}{\uparrow} \underbrace{\quad}_{positive}$