① (a) $\frac{dP}{dt} = \left(\frac{b}{100} - \frac{d}{100}\right) P + N$

(b) $0 = \frac{dP}{dt} = \left(\frac{b}{100} - \frac{d}{100}\right) P + N$

$\Rightarrow \quad N = -\left(\frac{b}{100} - \frac{d}{100}\right) P$

(c) $\frac{dP}{dt} = \left(\frac{4}{100} - \frac{9}{100}\right) P + 40$ and $P_0 = 1000$

$= -.05\, P + 40$

$= -.05\,(P - 800)$

Let $y = P - 800$.

Then $\frac{dy}{dt} = \frac{dP}{dt} = -.05\,(P-800) = -.05\, y$

$\Rightarrow \quad y(t) = y_0 e^{-.05t}$

$\Rightarrow \quad P(t) = y_0 e^{-.05t} + 800$

$1000 = P(0) = y_0 e^{0} + 800 \quad \Rightarrow \quad y_0 = 200$

$\Rightarrow \quad \boxed{P(t) = 200\, e^{-.05t} + 800}$

(d) Otters don't come in fractions.

② (a) $\frac{dP}{dt} = kP - C$

(b) Suppose $P(t) = P_0 e^{kt} - Ct$.

Then $\frac{dP}{dt} = k P_0 e^{kt} - C$

and $kP - C = k P_0 e^{kt} - kCt - C$.

not equal, so not a solution!

(c) Siphoning off C flies per day means that those flies leave the population (so the $-Ct$ make sense) but also that their offspring do not contribute to the population. Thus the siphoning affects the exponential growth of the population, but that isn't captured by the $P_0 e^{kt}$.

③ (a) The bank account bears 3% interest per year compounded continuously. Also, a total of \$3000 is withdrawn at a constant rate each year.

(b) $\frac{dM}{dt} = .03(40000) - 3000 = -1800$

Since the initial rate of growth is negative, the money added due to interest will never be enough to overcome the \$3000 withdrawals each year. So eventually the account will be depleted.

(c) $0 = 0.03M - 3000 \Rightarrow M = \$100{,}000$

An initial amount of \$100,000 results in a rate of change of \$0 per year.

(d) In addition to part (a), every year \$100 is added to the bank account for every year the account has been open. \$100 on 1st year, \$200 on 2nd year, etc.

(4) (a) When P is large, the $-.0001P^2$ term dominates the $0.03P$ term and so population decreases. Thus, too many lions results in overcrowding and population decline.

(b) $0 = \frac{dP}{dt} = 0.03P - 0.0001P^2$

$0 = 0.0001P(300 - P)$

$\Rightarrow$ $P = 0$ or 300 yield constant population

$\Rightarrow$ 300 is the carrying capacity of the savannah