SOLUTIONS

Math Xb Spring 2004
Worksheet: Introducing Differential Equations
May 3, 2004

1. Suppose the population of a bacteria colony can be modeled by the differential equation

$$\frac{dP}{dt} = \frac{P}{8}.$$

(a) Find the particular solution to this differential equation corresponding to the initial condition $P(0) = 1000$. Describe in words how the population changes over time.

$P_0 = 1000 \quad k = 1/8 \quad \Rightarrow \quad P(t) = 1000e^{t/8}$

The population grows exponentially from an initial population of 1000.

(b) Find the particular solution to this differential equation corresponding to the initial condition $P(10) = 1000$. Describe in words how the population changes over time.

$1000 = P(10) = P_0 e^{10/8}$

$P_0 = \frac{1000}{e^{5/4}} \approx 286.5$

$\Rightarrow P(t) \approx 286.5 e^{t/8}$

The population grows, in a sense, as fast as in part (a), but it starts smaller and only reaches 1000 after 10 time units.

2. Suppose the population of a bacteria colony can be modeled by the differential equation

$$\frac{dP}{dt} = kP.$$

Suppose also that when the colony has a population of 500 bacteria, the colony is growing at a rate of 100 bacteria per hour. If the colony has an initial population of 200 bacteria, find a formula for $P(t)$.

when $P = 500$, $\frac{dP}{dt} = 100$

$\Rightarrow 100 = k \cdot 500$

$k = 1/5$

$\Rightarrow P_0 = 200, \quad k = 1/5$

$\Rightarrow \boxed{P(t) = 200 e^{t/5}}$

3. Suppose a hot cup of coffee is taken out of a microwave and placed on a table in a 70° Fahrenheit to cool. At this time the temperature of the coffee is 180° F. When the temperature of the coffee is 150° F, the temperature of the coffee is decreasing at a rate of 10° F per minute. Determine when the temperature of the coffee will be 80° F.

T = coffee temp

70 = room temp

Newton's Law of Cooling

$\frac{dT}{dt} = k(T - 70)$

when $T = 150$, $\frac{dT}{dt} = -10$

$\Rightarrow -10 = k(150 - 70)$

$\Rightarrow k = -1/8$

$\Rightarrow T_0 = 180, \quad k = -1/8$

$\Rightarrow T(t) = (180 - 70)e^{-t/8} + 70$

$= 110e^{-t/8} + 70$

$80 = T(t) = 110e^{-t/8} + 70$

$10 = 110e^{-t/8}$

$\frac{1}{11} = e^{-t/8}$

$\ln 1/11 = -\frac{t}{8}$

$t = -8 \ln 1/11 = \ln 11^8 \approx 19.2$ min