Math Xb Spring 2004
Worksheet: Geometric Series
February 23, 2004

1. For each of the following geometric series, determine if the series converges or diverges. If it converges, find its sum.

   (a) $1 - 5 + 25 - 125 + 625 - \cdots$

   $r = -5$, so it diverges

   (b) $\frac{3}{4} + \frac{1}{2} + \frac{1}{3} + \frac{2}{9} + \cdots$

   $r = \frac{1}{2} / \frac{3}{4} = \frac{2}{3}$, so converges to $\frac{\frac{3}{4}}{1 - 2/3} = \frac{3 \cdot 3}{4} = 9/4$

   (c) $e + 1 + \frac{1}{e} + \frac{1}{e^2} + \cdots$

   $r = \frac{1}{e}$; so it converges to $\frac{e}{1 - 1/e} = \frac{e^2}{e-1}$

   (d) $7 - 14 + 28 - 56 + \cdots$

   $r = -2$, so it diverges

   (e) $1.2 + 0.96 + 0.768 + 0.6144 + 0.49152 + \cdots$

   $r = .8$, so it converges to $\frac{1.2}{1 - .8} = 6$

2. A manufacturing company that has just located in a small community will pay two million dollars per year in salaries. It has been estimated that 60% of these salaries will be spent in the local area, and 60% of the money spent will again change hands within the community. This process, called the *multiplier effect*, will be repeated ad infinitum. Find the total amount of local spending that will be generated by company salaries.

$$M = \sum_{i=1}^{\infty} 2{,}000{,}000 \cdot .60^{i} = \frac{2{,}000{,}000 \cdot .6}{.4} = 3{,}000{,}000$$

3. Find a rational number that corresponds to each of the following repeating decimals.

(a) $10.\overline{5}$

$$= 10 + .5 + \frac{.5}{10} + \frac{.5}{100} + \cdots = 10 + \sum_{i=0}^{\infty} \frac{.5}{10^{i}} = 10 + \frac{.5}{1 - 1/10} = \frac{95}{9}$$

(b) $5.4\overline{27}$

$$= 5.4 + \frac{27}{1000} + \frac{27}{100\,000} + \cdots = 5.4 + \sum_{i=1}^{\infty} \frac{27}{10^{2i+1}} = 5.4 + \frac{27/1000}{1 - 1/100} = \frac{597}{110}$$