SELECTED SOLUTIONS

Math Xb Spring 2004
Worksheet: Applications of Geometric Sums and Series
February 27, 2004

1. Amanda, at the young age of 9, has decided that she wants to be a doctor when she grows up. Her parents decide to set aside enough money in a bank account right now so that they will be able to withdraw \$10,000 every year for eight years beginning nine years from today. How much money should they put into an account with an annual interest rate of 6% compounded quarterly in order to do so?

2. Suppose you borrow some money at an interest rate of 6% compounded monthly. You begin paying back the money one year from today and make payments annually. You pay back the entire debt after 30 payments of \$1,000 each. How much money did you borrow?

3. In a pest eradication program, N sterilized male flies are released into the general population each day. It is estimated that 90% of these flies will survive a given day. If the long-range goal of the program is to keep 20,000 sterilized male flies in the population, how many flies should be released each day?

4. A certain drug has a half-life of about 2 hours in the bloodstream. The drug is formulated to be administered in doses of D milligrams every 4 hours. A level of more than 500 milligrams of the drug in the bloodstream is considered to be dangerous. Find the largest possible dose that can be given repeatedly over a long period of time.

5. A rubber ball is dropped from a height of 60 feet. If it rebounds approximately two-thirds the distance after each fall, find the total distance the ball travels.

6. The bob of a pendulum swings through an arc 24 centimeters long on its first swing. If each successive swing is approximately five-sixths the length of the preceding swing, find the total distance the bob travels.

(2) $M_t \left(1 + \frac{.06}{12}\right)^{12t} = 1000$

M_t: amount borrowed now; $\left(1 + \frac{.06}{12}\right)^{12t}$: interest applied for t years; 1000: payoff t years from now

$$\Rightarrow M_t = \frac{1000}{\left(1 + \frac{.06}{12}\right)^{12t}}$$

Want to find $M_1 + M_2 + \dots + M_{30}$

$$= \sum_{t=1}^{30} M_t = \sum_{t=1}^{30} \frac{1000}{\left(1 + \frac{.06}{12}\right)^{12t}} = \sum_{t=1}^{30} 1000\left[\left(1 + \frac{.06}{12}\right)^{-12}\right]^t$$

$$\approx \sum_{t=1}^{30} 1000\,(.9419)^t = \frac{1000(.9419)^1 - 1000(.9419)^{31}}{1 - .9419}$$

$$\approx \boxed{\$13,520.33}$$

④ amount after 0 hrs = D

amount after 4 hrs = $D + \frac{1}{4}D$

↑ from now ↑ from 4 hrs ago

amount after 8 hrs = $D + \frac{1}{4}D + \left(\frac{1}{4}\right)^2 D$

↑ from now ↑ from 4 hrs ago ↑ from 8 hrs ago

amount after 16 hrs = $D + \frac{1}{4}D + \left(\frac{1}{4}\right)^2 D + \left(\frac{1}{4}\right)^3 D$

amount after "∞ hrs"

$= D + \frac{1}{4}D + \left(\frac{1}{4}\right)^2 D + \cdots$

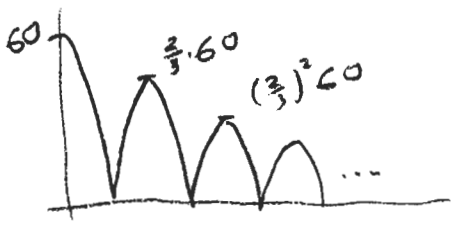
$= \sum_{n=0}^{\infty} D\left(\frac{1}{4}\right)^n = \frac{D}{1-\frac{1}{4}} = \frac{4}{3}D$

want $\frac{4}{3}D \leq 500$

$D \leq \frac{3}{4} \cdot 500 = 375$

375 mg

⑤



distance $= 60 + 2 \cdot \frac{2}{3}60 + 2 \cdot \left(\frac{2}{3}\right)^2 60$

$+ 2 \cdot \left(\frac{2}{3}\right)^3 60 + \cdots$

$= 60 + \sum_{n=1}^{\infty} 2 \cdot \left(\frac{2}{3}\right)^n 60$

$= 60 + \sum_{n=1}^{\infty} 120\left(\frac{2}{3}\right)^n$

$= 60 + \frac{120\left(\frac{2}{3}\right)}{1-\frac{2}{3}} = 60 + \frac{80}{1/3}$

$= 60 + 240 =$ **300 ft**