Problem 2.

(a) $F(t) = F_0(1-0.1)^t = F_0(0.9)^t$. To find the number of years it will take for half of the forestland to be destroyed, we find the value of t for which $0.5F_0 = F_0(0.9)^t$. Solving this equation, we have $0.5 = (0.9)^t$ $\Rightarrow \ln 0.5 = t \ln 0.9 \Rightarrow t = \frac{\ln 0.5}{\ln 0.9} \approx 6.58$ years.

(b) $F(t) = F_0 e^{-0.1t}$. To find the number of years it will take for half of the forestland to be destroyed, we find the value of t for which $0.5F_0 = F_0 e^{-0.1t}$. Solving this equation, we have $0.5 = e^{-0.1t} \Rightarrow \ln 0.5 = -0.1t$ $\Rightarrow t = -\frac{\ln 0.5}{0.1} \approx 6.93$ years.

(c) $\frac{dF}{dt} = -0.1F_0 e^{-0.1t} = -0.1F$. The sign of the proportionality constant is negative because the amount of forestland is decreasing.

Problem 3.

(a) $y(t) = Ce^{3t}$. $5 = Ce^{(0)t} \Rightarrow C = 5 \Rightarrow y(t) = 5e^{3t}$.

(b) $y(x) = Ce^{-0.01x}$. $1 = Ce^{-0.01(2)} \Rightarrow C = e^{0.02} \Rightarrow y(x) = e^{0.02}e^{-0.01x} = e^{0.02-0.01x}$.

(c) $w(s) = Ce^{w}$. $\pi = Ce^0 \Rightarrow C = \pi \Rightarrow w(s) = \pi e^s$.

Problem 6.

(a), (c) are solutions. For (a), $\dfrac{dy}{dt} = e^t = (e^t+2)-2 = y-2$. For (c), $\dfrac{dy}{dt} = Ce^t = (Ce^t+2)-2 = y-2$.

Problem 11.

$$\frac{dP}{dt} = 0.02P - 1000.$$

Problem 16.

(a) $\frac{dS}{dt} = kS(800-S)$ where k is a positive constant of proportionality.

(b) We maximize the quadratic function $R(S) = kS(800-S) = -kS^2 + 800kS$. Now $\dfrac{dR}{dS} = -2kS + 800k = -2k(S-400)$. $R(S)$ is maximized when $\dfrac{dR}{dS} = 0$, which occurs when $S = 400$. At this time, 400 students are sick.