

$$3. x^2 + y^2 = 169 \Rightarrow 2x \frac{dx}{dt} + 2y \frac{dy}{dt} = 0 \Rightarrow \frac{dx}{dt} = -\frac{y}{x} \frac{dy}{dt}.$$

(a) At point  $(-5, 12)$ ,  $\frac{dy}{dt} = -3$  so the bug is going counterclockwise.

$$(b) \frac{dx}{dt} \Big|_{(-5, 12)} = -\frac{12}{-5}(-3) = -\frac{36}{5} \frac{\text{units}}{\text{sec}}.$$

$$4) V = \frac{4}{3} \pi r^3 \Rightarrow \frac{dV}{dt} = \frac{4}{3} \pi 3r^2 \frac{dr}{dt} \Rightarrow \frac{dr}{dt} = \frac{\frac{dV}{dt}}{4\pi r^2}$$

$$(a) \frac{dr}{dt} \Big|_{r=15} = \frac{-0.5}{4\pi(15)^2} = \frac{-1}{1800\pi} \frac{\text{cm}}{\text{hr}}$$

$$(b) \frac{dr}{dt} \Big|_{r=15/2} = \frac{-0.5}{4\pi(15/2)^2} = \frac{-1}{450\pi} \frac{\text{cm}}{\text{hr}}$$

$$(c) D = 2r \Rightarrow \frac{dD}{dt} \Big|_{r=15/2} = 2 \frac{dr}{dt} \Big|_{r=15/2} = \frac{-1}{4\pi(15/2)^2} = \frac{-1}{225\pi} \frac{\text{cm}}{\text{hr}}$$

$$(d) 12\pi = 2\pi r \Rightarrow r = 6 \Rightarrow \frac{dr}{dt} \Big|_{r=6} = \frac{-0.5}{4\pi(6)^2} = \frac{-1}{288\pi} \frac{\text{cm}}{\text{hr}}$$

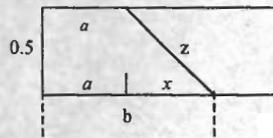
$$5. (a) V = \pi r^2 h = \pi(3)^2 h = 9\pi h$$

(b)  $\frac{dV}{dh} = 9\pi$  is constant as a change in  $h$  will cause a similar change in volume.

$$(c) \text{ i. } \frac{dV}{dh} = 9\pi \frac{dh}{dt}, \frac{dV}{dt} = 9\pi(-\frac{1}{2}) = -\frac{9}{2}\pi \frac{ft^3}{hr}.$$

$$\text{ ii. } -3 = 9\pi \frac{dh}{dt} \Rightarrow \frac{dh}{dt} = -\frac{1}{3\pi} \frac{ft}{hr}$$

9. Let  $a$  and  $b$  denote the distance down the road of the two cars. Assume  $b > a$  then let  $x$  denote how much larger  $b$  is than  $a$ . If  $z$  is the distance between the two cars



$$\text{ then } z^2 = (0.5)^2 + x^2$$

$$\Rightarrow z^2 = (0.5)^2 + (b-a)^2$$

$$(a) 2z \frac{dz}{dt} = 2(b-a) \left( \frac{db}{dt} - \frac{da}{dt} \right) \Rightarrow \frac{dz}{dt} = \frac{(b-a)}{z} \left( \frac{db}{dt} - \frac{da}{dt} \right) \quad \text{in 2 hrs } a = 60\text{m and } b = 80\text{m}$$

$$z^2 = 0.25 + (80-60)^2 = 0.25 + 400 = 400.25 \quad \text{so } \frac{dz}{dt} = \frac{20}{\sqrt{400.25}} (40-30) \approx 9.9969 \frac{\text{m}}{\text{hr}}$$

$$(b) z = 40 \Rightarrow 1600 = 0.25 + (b-a)^2 \Rightarrow (b-a)^2 = 1599.75 \Rightarrow b-a \approx 39.9969$$

$$\text{ so } \frac{dz}{dt} = \frac{39.9969}{40} (10) \approx 9.9992 \frac{\text{m}}{\text{hr}}$$

$$(c) z^2 = (0.25) + (b+a)^2. \quad \text{In 2 hrs, } z^2 = (0.25) + (140)^2 = (0.25) + 19600 = 19600.25$$

$$\frac{dz}{dt} = \frac{b+a}{z} \left( \frac{db}{dt} + \frac{da}{dt} \right) = \frac{140}{\sqrt{19600.25}} (40+30) \approx 69.9996 \frac{\text{m}}{\text{hr}}$$

$$- 14. \text{ By similar triangles } \frac{x+l}{14} = \frac{l}{5.5} \Rightarrow 5.5x + 5.5l = 14l \Rightarrow 8.5l = 5.5x$$

$$\Rightarrow 8.5 \frac{dl}{dt} = 5.5 \frac{dx}{dt} \Rightarrow \frac{dl}{dt} = \frac{5.5(4)}{8.5} = \frac{44}{17} \approx 2.588 \frac{\text{ft}}{\text{sec}}$$