20.3 Inverse Trigonometric Functions

1. (a) $\frac{\pi}{2}$ (b) $\frac{\pi}{4}$ (c) $-\frac{\pi}{2}$

(d) π (e) $-\frac{\pi}{6}$ (f) $\frac{2\pi}{3}$

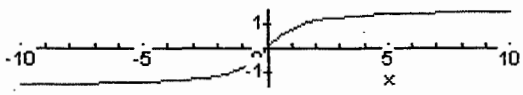
(g) $\frac{\pi}{3}$ (h) $(\pi)^2 + (-\frac{\pi}{4})^2 = \frac{17\pi^2}{16}$

2. (a) Find the arc length in quadrant I associated with a vertical displacement of 0.8 $\theta \approx 0.93$

(b) Find the arc length in quadrant I associated with a horizontal displacement of 0.8. $\theta \approx 0.93$

(c) Find the arc length in quadrant I associated with a y to x ratio of 2 to 1, $\theta \approx 1.1$

3. (a) $y = \tan^{-1} x$
y' pos. for all x



(b) odd (c) even

7. (a) $\frac{3}{5}$ (b) $\sqrt{3}$ (c) $\sqrt{1-x^2}$, $(-\frac{\pi}{2} \le \theta \le 0)$ (d) $\frac{w}{\sqrt{r^2-w^2}}$, $(0 \le \theta \le \frac{\pi}{2})$

20.4 Solving Trigonometric Equations

1. (a) $\cos x = \pm\frac{\sqrt{3}}{2} \Rightarrow x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}$, or $\frac{11\pi}{6}$

(b) $2u^2 - u - 1 = 0 \Rightarrow u = -\frac{1}{2}, 1 \Rightarrow \sin x = -\frac{1}{2}, \sin x = 1 \Rightarrow x = \frac{7\pi}{6}, \frac{11\pi}{6}$, or $\frac{\pi}{2}$

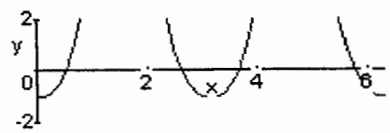
(c) $x = \sin^{-1}\frac{2}{3} \approx .7297$ or $\pi - \sin^{-1}\frac{2}{3} \approx 2.4119$

2. (a) $\sin x = \frac{1}{2}, \Rightarrow x = \frac{\pi}{6} + k2\pi, \frac{5\pi}{6} + k2\pi$, $\sin x = -\frac{1}{2}, \Rightarrow x = -\frac{\pi}{6} + k2\pi, -\frac{5\pi}{6} + k2\pi$, $k \in \mathbf{Z}$
combined we get $x = \frac{\pi}{6} + k\pi$, $k \in \mathbf{Z}$ *or* $-\frac{\pi}{6} + k\pi$, $k \in \mathbf{Z}$

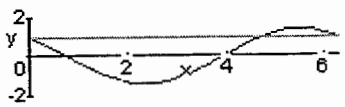
(b) $u^2 + 2u + 1 = 0 \Rightarrow u + 1 = 0 \Rightarrow \cos x = u = -1 \Rightarrow x = \pi + k2\pi$, $k \in \mathbf{Z}$

(c) $u^2 + 4u + 3 = 0 \Rightarrow u = -1, -3 \Rightarrow \cos x = u = -1 \Rightarrow x = \pi + k2\pi$, $k \in \mathbf{Z}$ ($\cos x = -3$ never)

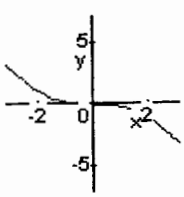
8. $\tan^2 x = \frac{1}{3} \Rightarrow \tan x = \pm\frac{1}{\sqrt{3}}$
$\Rightarrow x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$



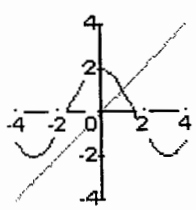
13. $1 - \sin^2 x = 1 + 2\sin x + \sin^2 x \Rightarrow$
$2\sin x(\sin x + 1) = 0 \Rightarrow \sin x = 0$ or $\sin x = -1$
$x = 0, \pi, 2\pi$ *or* $\frac{3\pi}{2}$ (only π doesn't work)
So solutions set is $\{0, 2\pi, \frac{3\pi}{2}\}$



14.(a) graphing $(\sin x - x)$ yields only one zero of $x = 0$.



(b) graphing $2\cos x$ and x yields only one place where they cross. $x \approx 1$



(c) graphing $\sin x - \frac{x}{3}$ yields three zeros of $x \approx 0, -2.3, 2.3$