20.5 Applying Trigonometry to a General Triangle: The Law of Cosines and the Law of Sines

1. (a) $\frac{1}{2}(3)(4)\sin 120^\circ = \frac{1}{2}(3)(4)\frac{\sqrt{3}}{2} = 3\sqrt{3} \approx 5.196$

(b) $\frac{1}{2}(10)(12)\sin 70^\circ \approx \frac{1}{2}(10)(12)(.9497) = 56.38$

(c) $\frac{1}{2}(2)(2)\sin 60^\circ = \frac{1}{2}(2)(2)\frac{\sqrt{3}}{2} = \sqrt{3} \approx 1.73$

2. $\sin\theta = \sin(\pi - \theta) = \frac{4}{5}$, $\quad \cos\theta = -\cos(\pi - \theta) = -\frac{3}{5}$, $\quad \tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{4}{5} \div (-\frac{3}{5}) = -\frac{4}{3}$

4. (a) Let $A = 25^\circ$, $C = 30^\circ$ and $c = 4$.

Then $B = (180^\circ - (25^\circ + 30^\circ)) = 125^\circ$.

$$\frac{a}{\sin(25^\circ)} = \frac{4}{\sin(30^\circ)} \Rightarrow a = \frac{4\sin(25^\circ)}{\sin(30^\circ)} \approx 3.4$$

$$\frac{b}{\sin(125^\circ)} = \frac{4}{\sin(30^\circ)} \Rightarrow b = \frac{4\sin(125^\circ)}{\sin(30^\circ)} \approx 6.6$$

(b) Let $a = 2$, $b = 3$ and $c = 4$.

$$2^2 = 3^2 + 4^2 - 2(3)(4)\cos(A) \Rightarrow \cos(A) = \frac{-21}{-24} = \frac{7}{8} \Rightarrow A \approx 29.0^\circ$$

$$3^2 = 2^2 + 4^2 - 2(2)(4)\cos(B) \Rightarrow \cos(B) = \frac{-11}{-16} = \frac{11}{16} \Rightarrow B \approx 46.6^\circ$$

$$C = 180^\circ - 29^\circ - 46.6^\circ = 104.4^\circ$$

(c) Let $a = 6$, $b = 10$ and $B = 120^\circ$.

$$\frac{\sin(120^\circ)}{10} = \frac{\sin(A)}{6} \Rightarrow \sin(A) \approx .5196 \Rightarrow A \approx 31.3^\circ$$

$$C = 180^\circ - 120^\circ - 31.3^\circ = 28.7^\circ$$

$$\frac{c}{\sin(28.7^\circ)} = \frac{10}{\sin(120^\circ)} \Rightarrow c \approx 5.5$$

(d) Let $a = 3$, $b = 4$ and $C = 25^\circ$

$$c^2 = 3^2 + 4^2 - 2(3)(4)\cos(25^\circ) \approx 3.249 \Rightarrow c \approx 1.8$$

$$\frac{\sin(25^\circ)}{1.8} = \frac{\sin(A)}{3} \Rightarrow \sin(A) \approx .7034 \Rightarrow A \approx 44.7^\circ \text{ (must be acute)}$$

$$B = 180^\circ - 25^\circ - 44.7^\circ = 110.3^\circ$$

7. $PQ^2 = 2.5^2 + 3^2 - 2(2.5)(3)\cos(30^\circ) \approx 2.26 \Rightarrow PQ \approx 1.5\, mi$.

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2. (a) $\cos(\frac{\pi}{12}) = \cos(\frac{\pi}{4}-\frac{\pi}{6}) = \cos\frac{\pi}{4}\cos\frac{\pi}{6}+\sin\frac{\pi}{4}\sin\frac{\pi}{6} = \frac{1}{\sqrt{2}}\frac{\sqrt{3}}{2}+\frac{1}{\sqrt{2}}\frac{1}{2} = \frac{\sqrt{2}}{4}(\sqrt{3}+1)$

(b) $\sin(\frac{-\pi}{12}) = \sin(\frac{\pi}{6}-\frac{\pi}{4}) = \sin\frac{\pi}{6}\cos\frac{\pi}{4}-\cos\frac{\pi}{6}\sin\frac{\pi}{4} = \frac{1}{2}\frac{1}{\sqrt{2}}-\frac{\sqrt{3}}{2}\frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{4}(1-\sqrt{3})$

(c) $\tan(\frac{\pi}{12}) = \dfrac{\sin\frac{\pi}{12}}{\cos\frac{\pi}{12}} = \dfrac{-\sin(-\frac{\pi}{12})}{\cos\frac{\pi}{12}} = \dfrac{-\frac{\sqrt{2}}{4}(1-\sqrt{3})}{\frac{\sqrt{2}}{4}(\sqrt{3}+1)} = \dfrac{(-1+\sqrt{3})}{(\sqrt{3}+1)}\dfrac{(\sqrt{3}-1)}{(\sqrt{3}-1)} = 2-\sqrt{3}$

3. $\sin(x-2y) = \sin x\cos 2y - \cos x\sin 2y = \sin x(\cos^2 y - \sin^2 y) - \cos x(2\sin y\cos y)$
$\cos(x-2y) = \cos x\cos 2y + \sin x\sin 2y = \cos x(\cos^2 y - \sin^2 y) + \sin x(2\sin y\cos y)$

7. (a) $\cos^2 x - \cos 2x = 0 \Rightarrow \cos^2 x - (\cos^2 x - \sin^2 x) = 0 \Rightarrow \sin^2 x = 0 \Rightarrow x = k\pi$

(b) $\sin x\cos x = \sqrt{3} \Rightarrow \sin(2x) = 2\sqrt{3} > 1$ which is impossible, hence no solution.

(c) $\sin^2 x - \cos 2x = 0 \Rightarrow \sin^2 x - (1-2\sin^2 x) = 0 \Rightarrow \sin^2 x = \frac{1}{3} \Rightarrow \sin x = \pm\sqrt{\frac{1}{3}}$
$\Rightarrow x = \sin^{-1}\sqrt{\frac{1}{3}} \approx 0.615$ or $\pi \pm \sin^{-1}\sqrt{\frac{1}{3}} \approx 2.526, 3.757$ or $2\pi - \sin^{-1}\sqrt{\frac{1}{3}} \approx 5.668$

(d) $-\cos^2 x + \frac{1}{2}\sin x + 1 = 0 \Rightarrow -(1-\sin^2 x) + \frac{1}{2}\sin x + 1 = 0 \Rightarrow (\sin x)(\sin x + \frac{1}{2}) = 0$
$\Rightarrow \sin x = 0$ *or* $\sin x = -\frac{1}{2} \Rightarrow x = 0, \pi, 2\pi$ *or* $x = \frac{7\pi}{6}, \frac{11\pi}{6}$

8. $\cos(3x) = \cos(2x+x) = \cos(2x)\cos x - \sin(2x)\sin x$
$= (2\cos^2 x - 1)\cos x - 2\sin x\cos x\sin x$
$= 2\cos^3 x - \cos x - 2\cos x\sin^2 x = 2\cos^3 x - \cos x - 2\cos x(1-\cos^2 x)$
$= 2\cos^3 x - \cos x - 2\cos x + 2\cos^3 x = 4\cos^3 x - 3\cos x$